



Univerzita Hradec Králové  
Filozofická fakulta

# E-learningový kurz

Modern quantitative methods  
and shape analysis in archaeology



EVROPSKÁ UNIE  
Evropské strukturální a investiční fondy  
Operační program Výzkum, vývoj a vzdělávání



MINISTERSTVO ŠKOLSTVÍ,  
MLÁDEŽE A TĚLOVÝCHOVY

Tento materiál vznikl v rámci realizace projektu  
Strategický rozvoj Univerzity Hradec Králové,  
reg. č. CZ.02.2.69/0.0/0.0/16\_015/0002427.

# Outline analyses

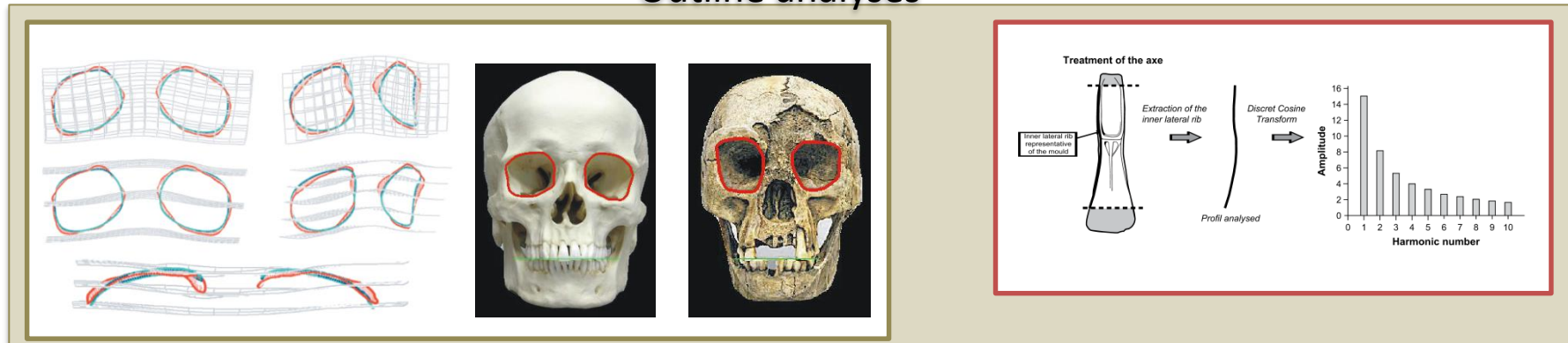


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## Landmarks analyses



## Outline analyses



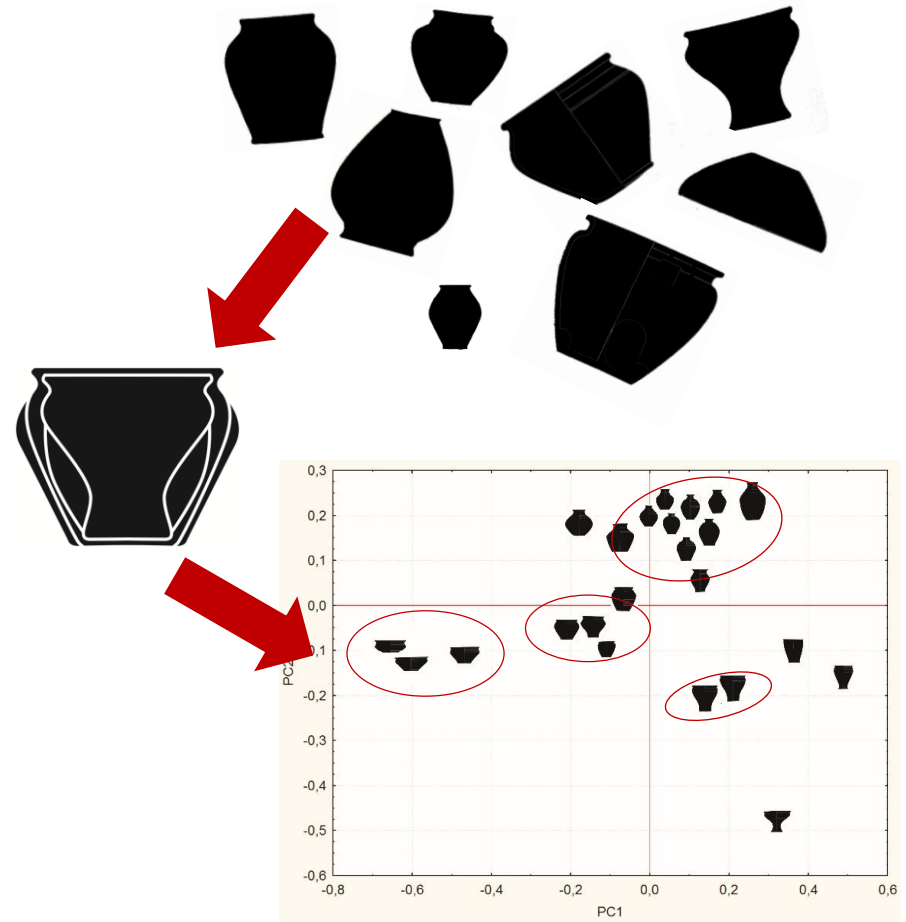
## 1) Data collection

## 2) Standardisation

(position, size and orientation)

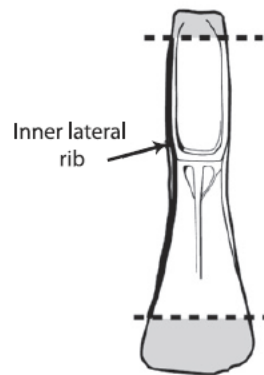
## 3) Calculation of shape variables

## 4) Data treatment and visualisation



## Semilandmarks (Open outlines)

xy-coordinates of points traced along an outline



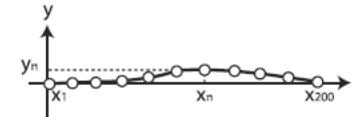
**1**  
Inner lateral rib,  
representative of the  
mould



**2**  
Drawing on  
tracing paper



**3**  
Scanning at  
300 dpi

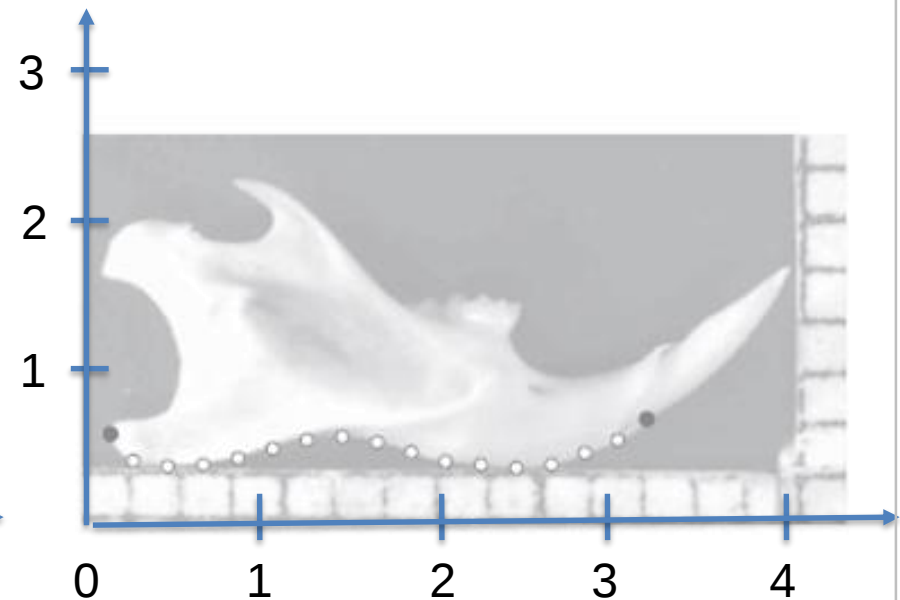
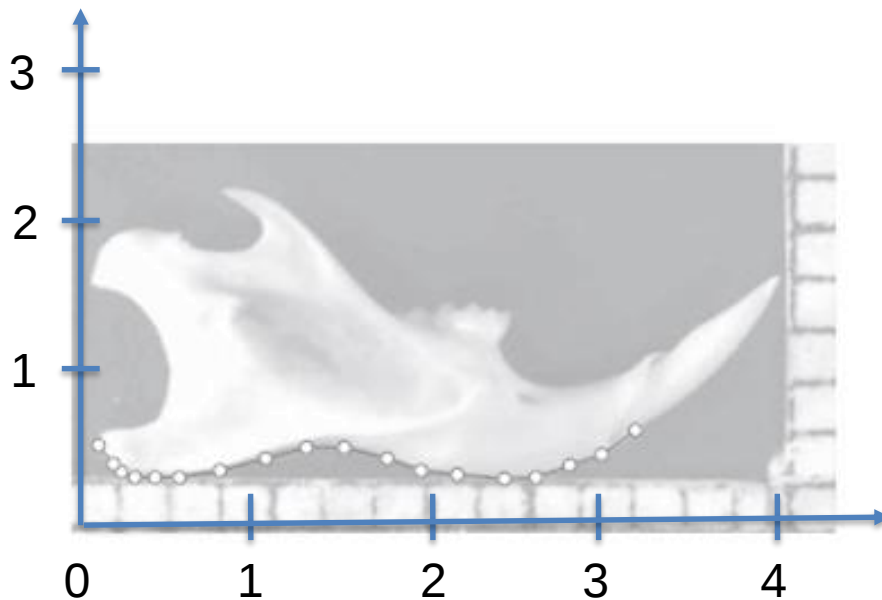


| X                | Y                |
|------------------|------------------|
| X <sub>1</sub>   | y <sub>1</sub>   |
| X <sub>2</sub>   | y <sub>2</sub>   |
| ...              | ...              |
| X <sub>n</sub>   | y <sub>n</sub>   |
| ...              | ...              |
| X <sub>200</sub> | y <sub>200</sub> |

**4**  
Sampling 200 equally spaced  
points along the chord

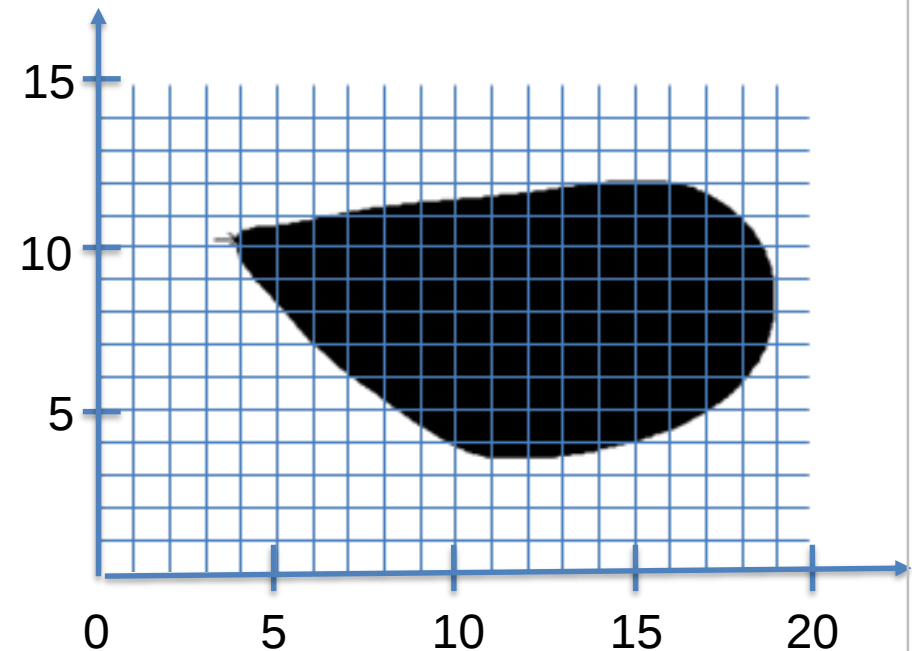
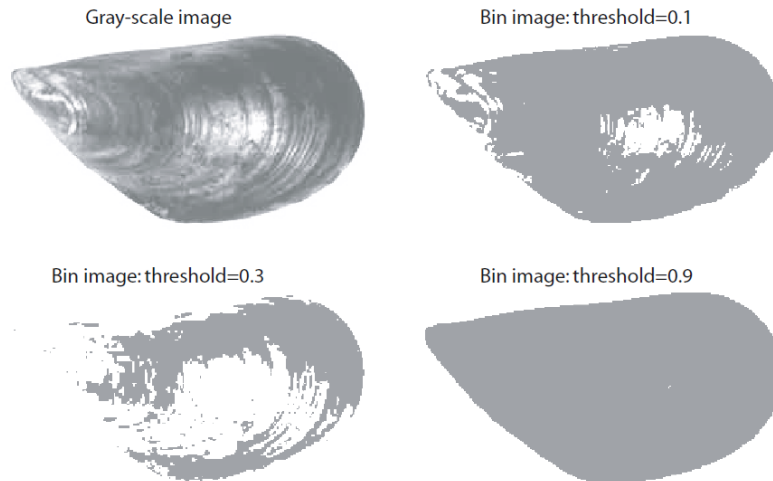
## Semilandmarks (Open outlines)

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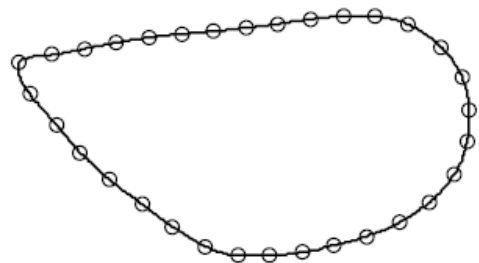
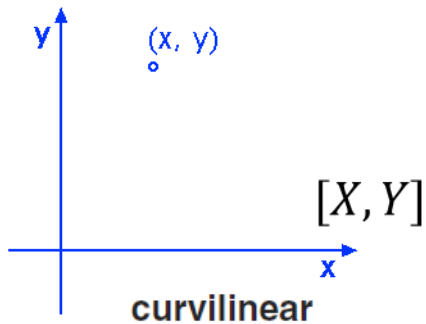


## Semilandmarks (Closed outlines)

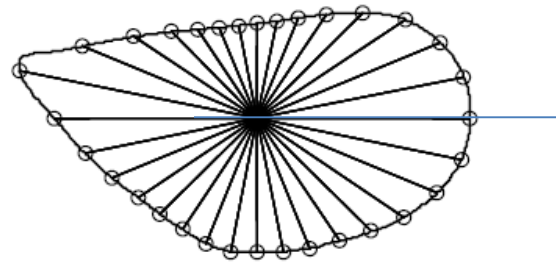
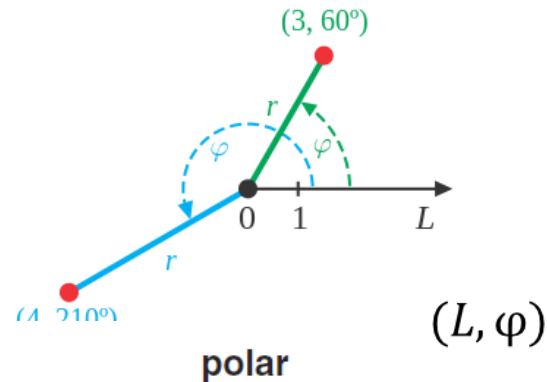
automatic xy coordinates extraction



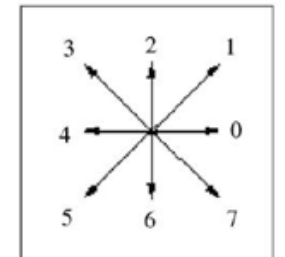
## Semilandmarks



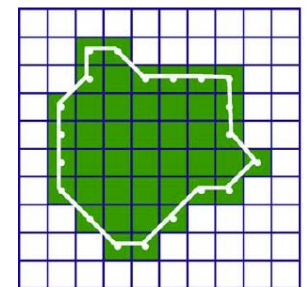
equally spaced segments



equally spaced angles



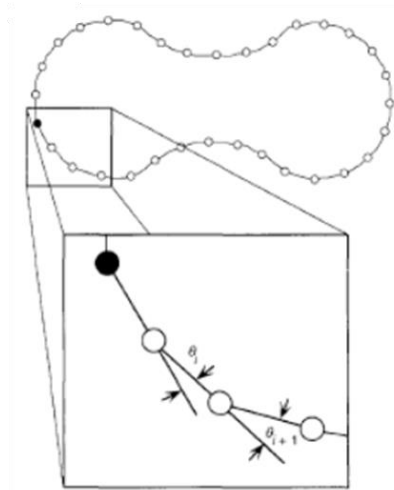
Chain code:  
071000667545543322212



chaincode

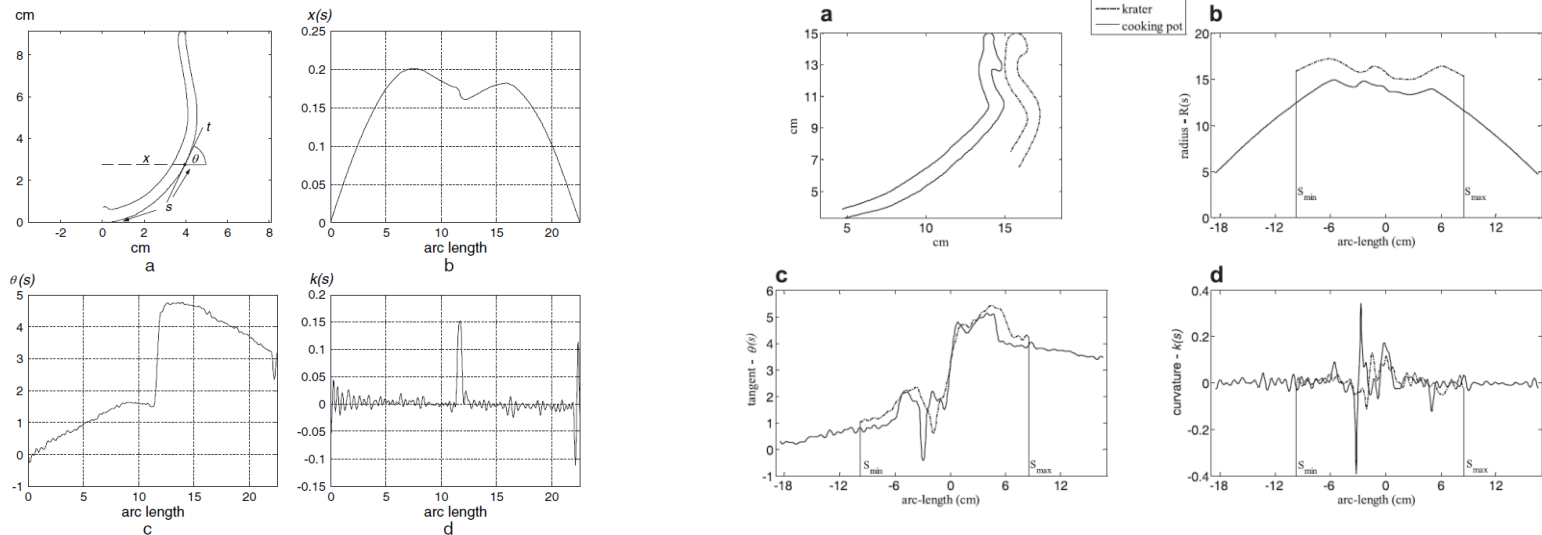
## Tangent angles

Curve is expressed as series of angles between semilandmarks Phi function (Zahn and Roskies shape function) => eigenshape analysis



## Radius, tangent and curvature function

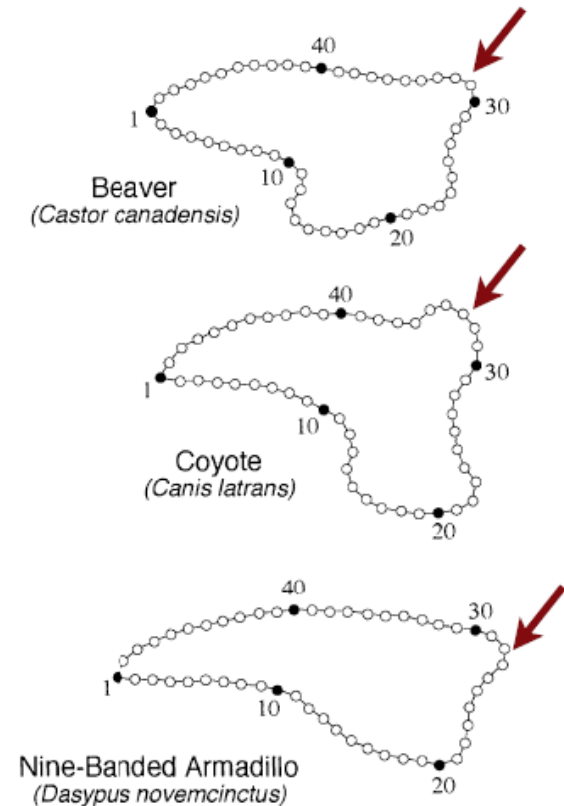
Artefacts (often pottery profiles) are sometimes expressed by **radius function**; **tangent/slope function** (1<sup>st</sup> derivate), and **curvature function** (2<sup>nd</sup> derivate)



## Problems with homology semilandmarks

It does not have effect **overall** similarities/differences between shapes

It may be hard to locate parts which are similar/different

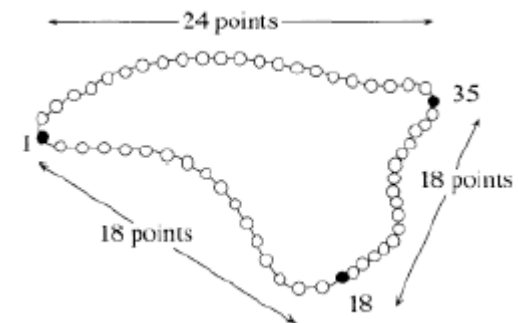
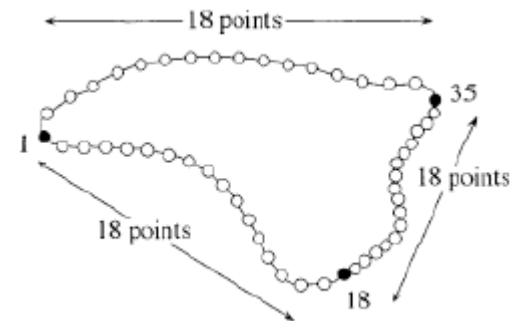


## (Partial) solution = Extended eigenshape

Landmarks for homologous points and semilandmarks between them

A) The same number of semilandmarks on each segment

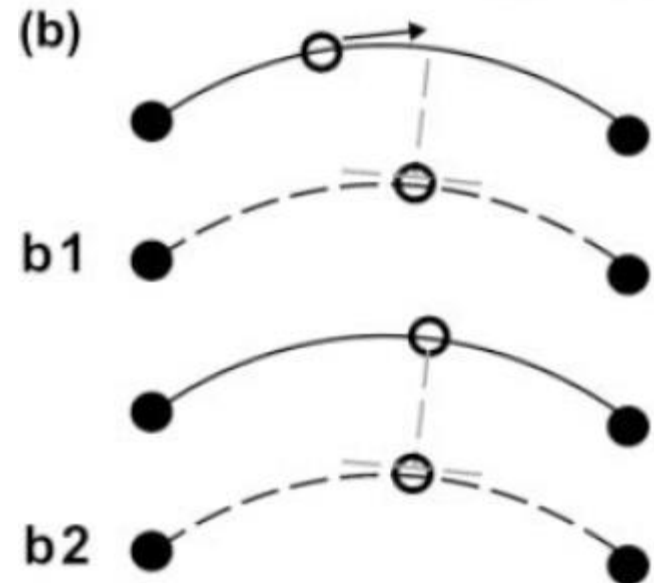
B) Number of landmarks in each segment varies based on its complexity



## (Partial) solution = Sliding semilandmarks

Semilandmarks are iteratively “sliding” along outline until the best correspondence is found

Finding minimums by using Procrustes distance or Bending energy



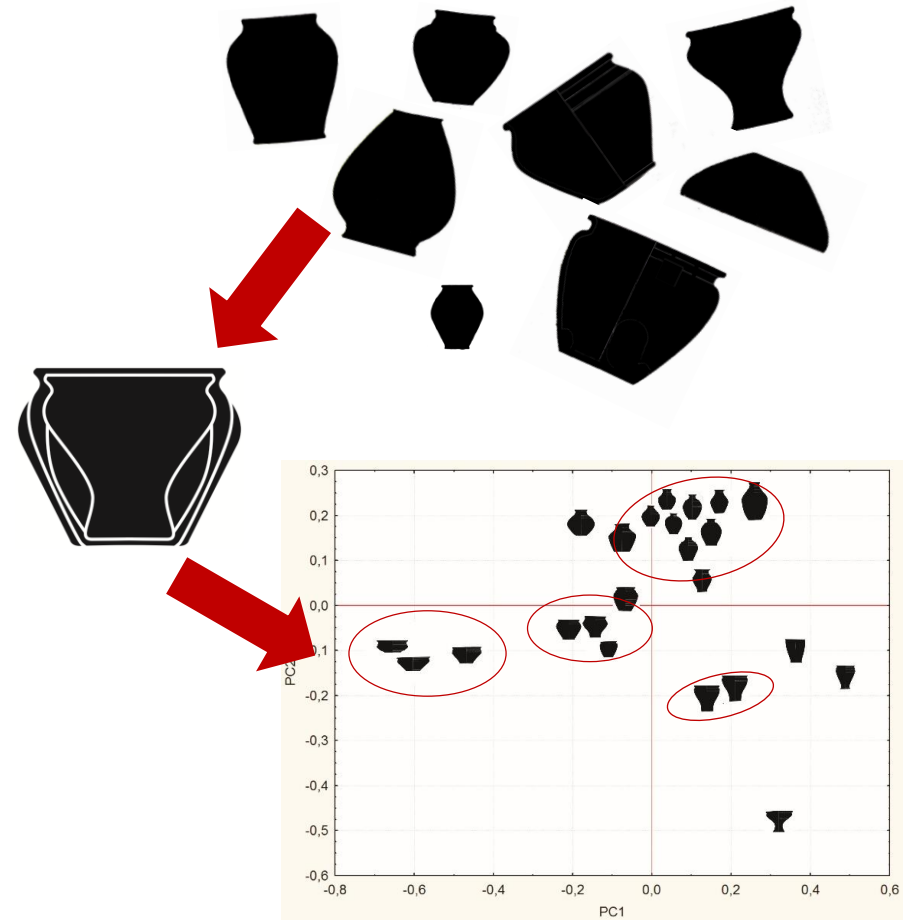
## 1) Data collection

## 2) Standardisation

(position, size and orientation)

## 3) Calculation of shape variables

## 4) Data treatment and visualisation



Outlines are standardised in the same way as landmarks

Functions representations are standardised by (i) the length of curvilinear abscissa, (ii) by arc-length, or (iii) by perimeter

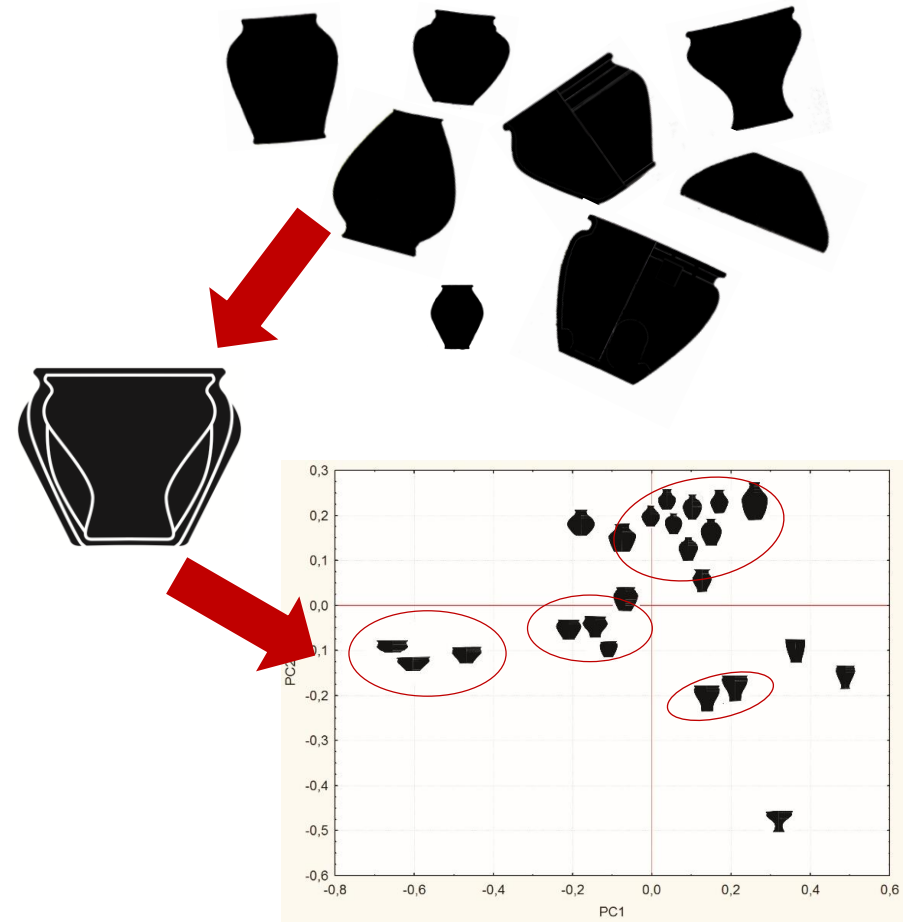
## 1) Data collection

## 2) Standardisation

(position, size and orientation)

## 3) Calculation of shape variables

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**Semilandmarks => Procrustes analysis**

**Sliding semilandmarks => Procrustes analysis**

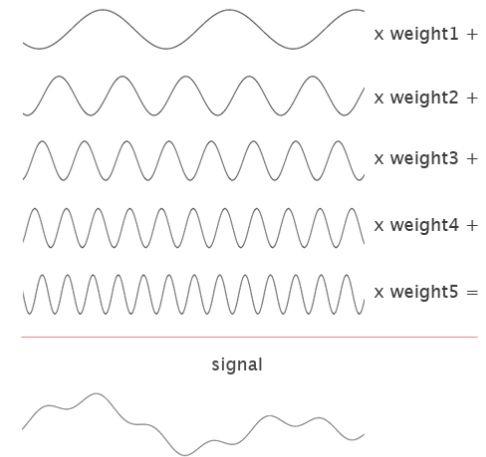
**Angles/Functions => directly**

## Discrete Cosine Transform (DCT)

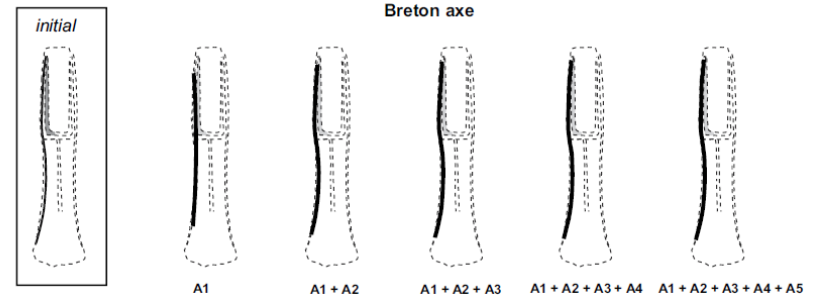
Uses harmonics (trigonometric cosine waves) to describe shape

### Principle:

- 1) Each outline can be viewed as a signal and can be decomposed into the sum of waves (**harmonics**)
- 2) Each wave is expressed by **2 coefficients**
- 3) If we sum the coefficients, we can reconstruct the original shape (**Inverse transform**)



$$S[k] = \sum_{n=0}^{N-1} c[k]s[n] \cos\left(\frac{(2n+1)k\pi}{2N}\right)$$



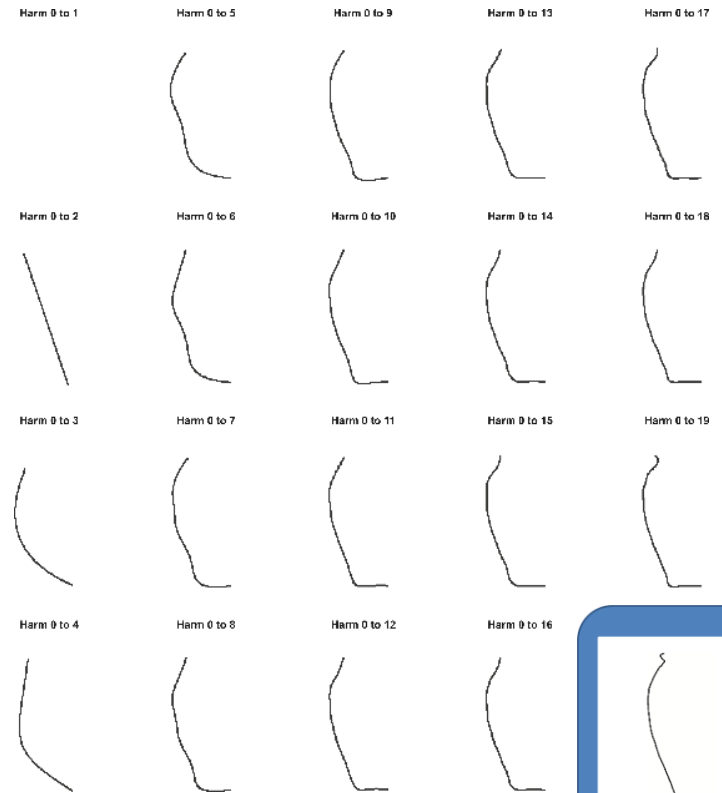
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## Elliptic Fourier Analysis (EFA)

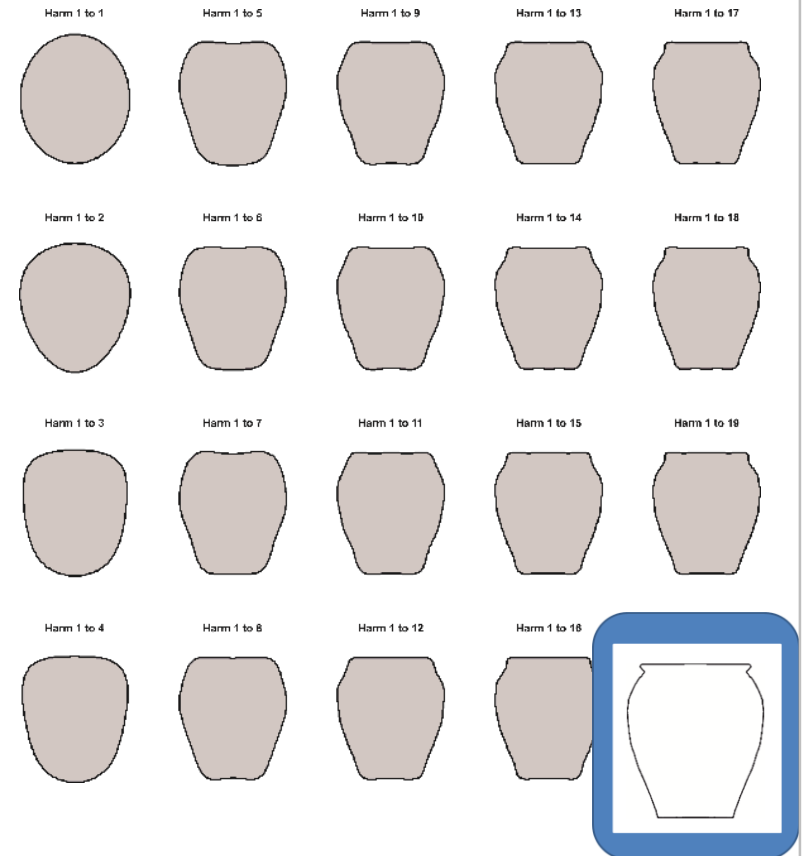
Uses harmonics (trigonometric sine and cosine waves) to describe shape

### Principle:

- 1) Each outline can be viewed as a signal and can be decomposed into the sum of waves (**harmonics**), representing **ellipses**
- 2) Each wave is expressed by **4 coefficients**
- 3) If we sum the coefficients, we can reconstruct the original shape (**Inverse Fourier transform**)

$$x(t) = a_0 + \sum_{j=1}^k a_j \cos\left(\frac{2j\pi t}{T}\right) + \sum_{j=1}^k b_j \sin\left(\frac{2j\pi t}{T}\right)$$

$$y(t) = c_0 + \sum_{j=1}^k a_j \cos\left(\frac{2j\pi t}{T}\right) + \sum_{j=1}^k d_j \sin\left(\frac{2j\pi t}{T}\right)$$



## Normalised EFA (NEFA)

Normalisation according to the first ellipse  
 First three coefficients become constant  
 Usually, we do not use 1<sup>st</sup> harmonic in calculations

## Nyquist criterion

Maximum number of harmonics equal half the number of coordinates

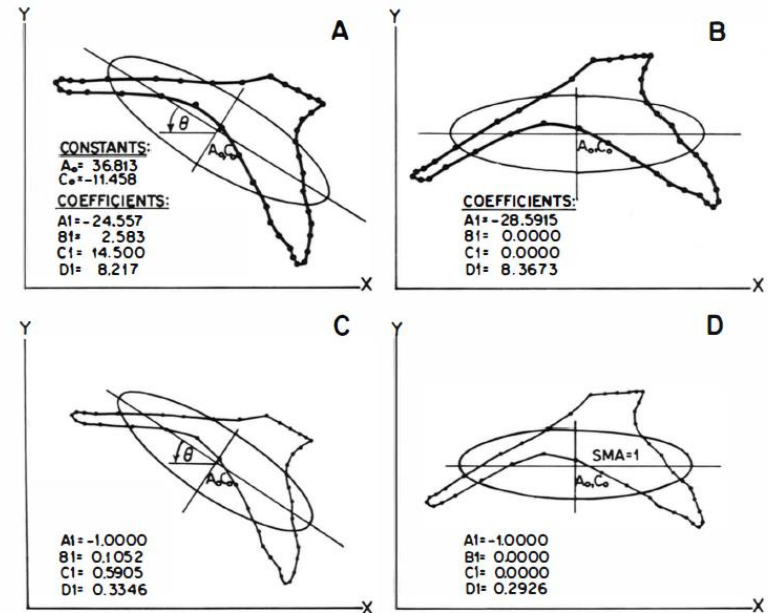


Fig. 5. Four theoretically possible normalizations. A: No positional-orientation and no size-standardization. B: Positional-orientation but no size-standardization. C: No positional-orientation but with size-standardization present. D: Both positional-orientation and size-standardization present. See text for further discussion.

## Bezier Polynomials

Fitting a curve to an open outline

A Bezier polynomial of degree  $q$  is a function that passes to a sequence of  $q + 1$  points (e.g. for 10 points we can have Bezier polynomial of degree 9)

Decomposition of original coordinates into two matrices:

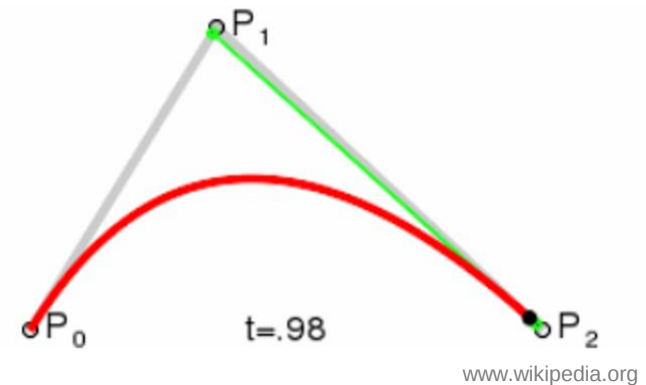
$$M = JB$$

$M$  is the original configuration

$J$  is a  $q \times q + 1$  matrix of Bezier coefficients

$B$  is a matrix of coordinates for the Bezier vertices

$B$  used as shape variables

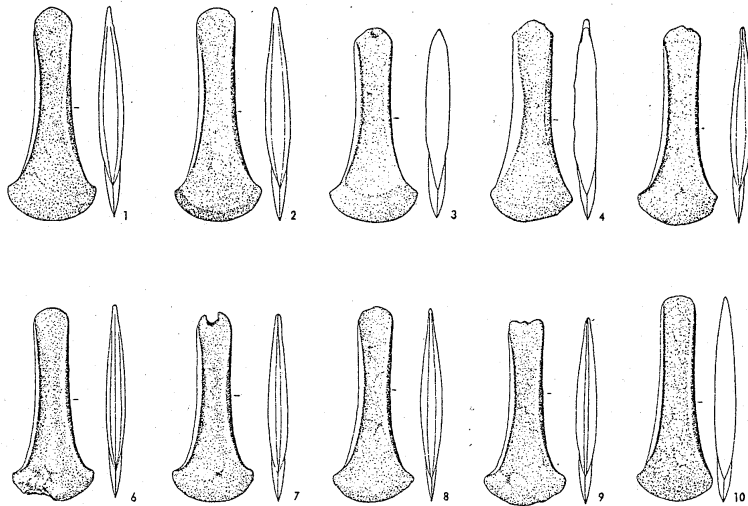


# Case study



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Operační program Výzkum, vývoj a vzdělávání

## Middle Bronze Age axes (1650-1350 BC)

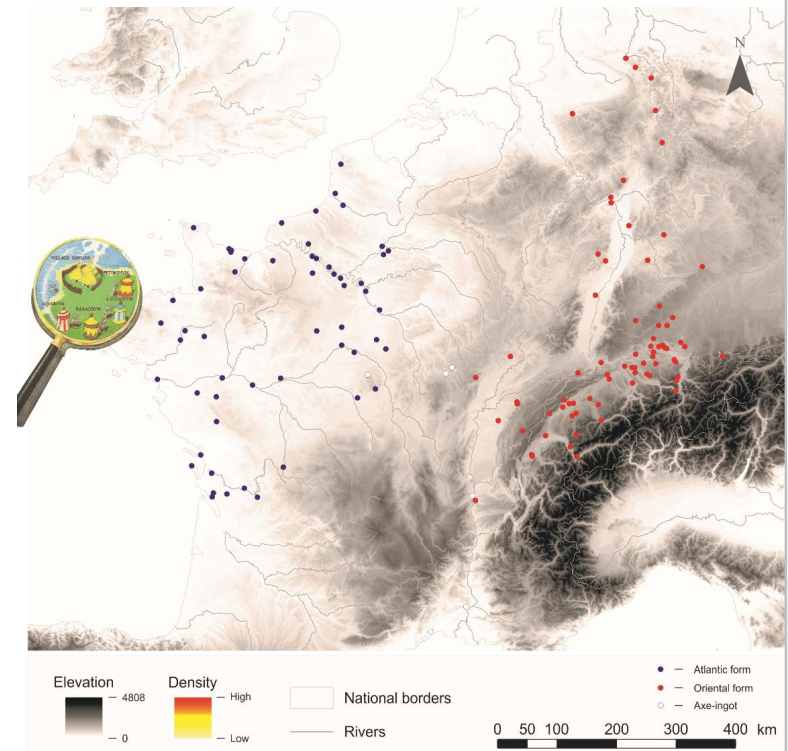
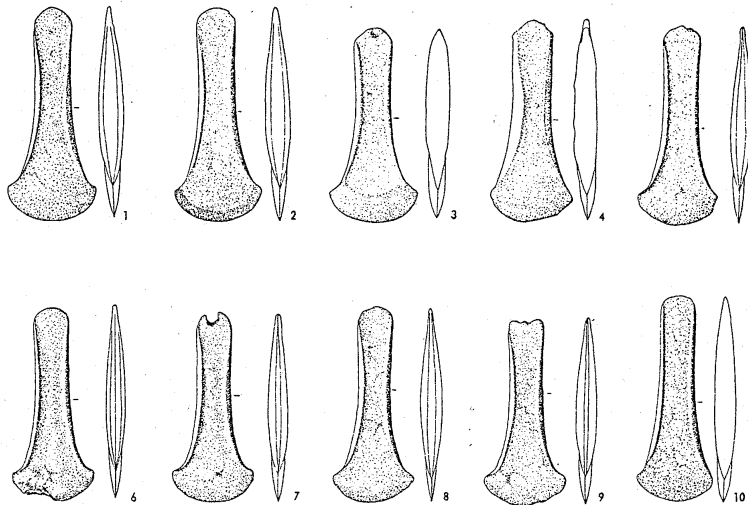


## Checking the current typology of flanged axes

Specific types (Neyruz, Salez, Möhlin, ...)

Atlantic vs Oriental type, „Axes-ingots“

Geometric morphometry (EFA)



## Choice of corpus

Entirely preserved

Not much used

A-priori classified (Atlantic/Oriental type)

Axe-ingots

## Studied corpus

286 axes from 132 sites

121 oriental forms

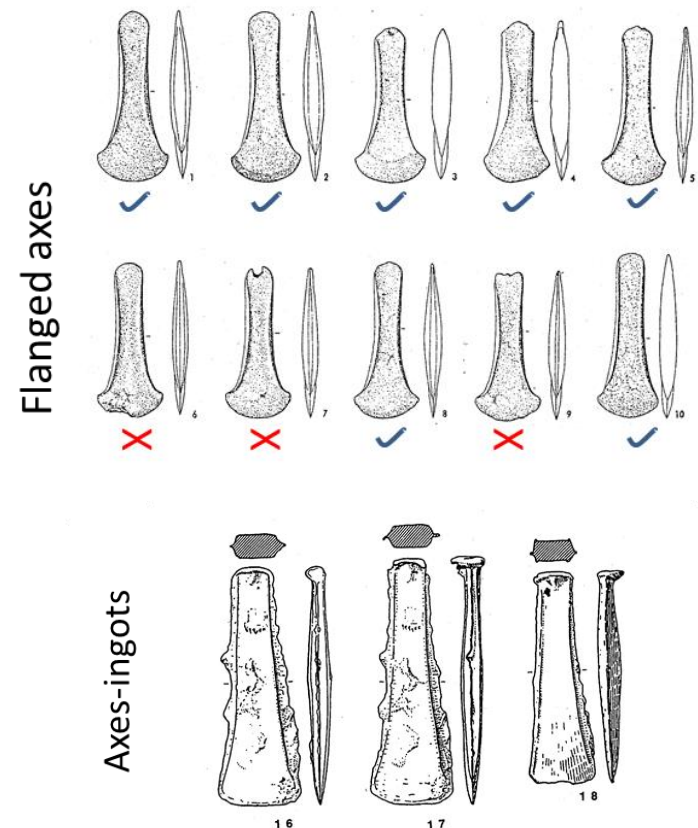
126 occidental forms

21 axes-ingots

## Axes-ingots

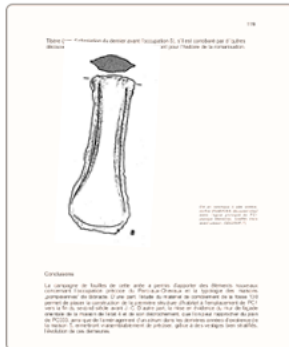
Similar to flanged axes

Residual parts on borders



## Morphometrics

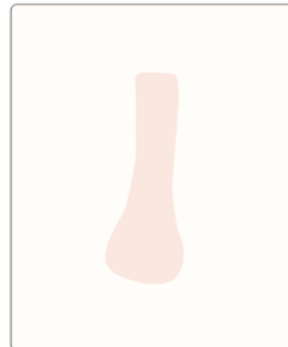
Original publication



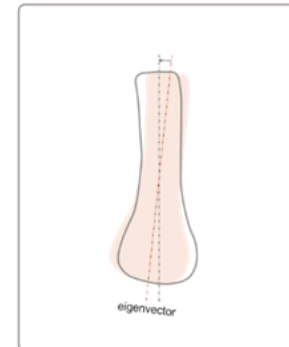
Retracing and scanning at 300 dpi



Extracting of silhoulett



Vertical orientation



Sampling 200 points



## EFA

$$x(t) = a_0 + \sum_{j=1}^k a_j \cos\left(\frac{2j\pi t}{T}\right) + \sum_{j=1}^k b_j \sin\left(\frac{2j\pi t}{T}\right)$$

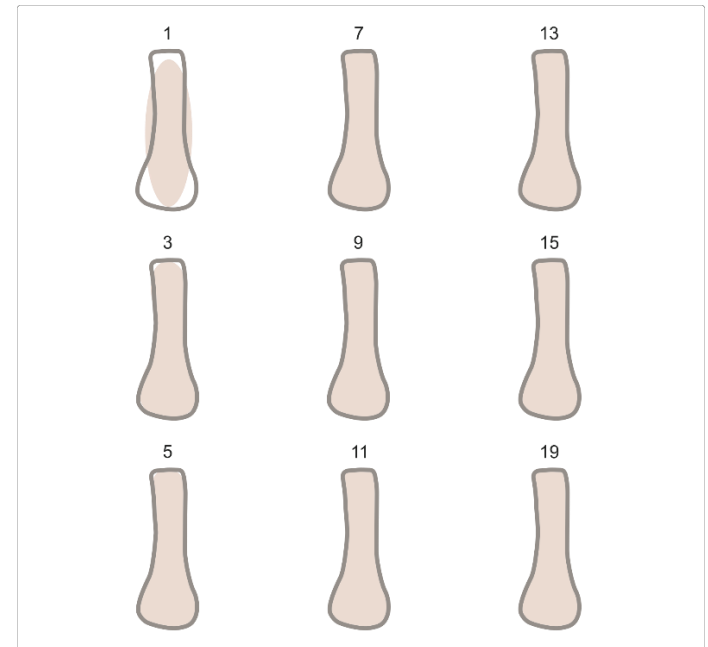
$$y(t) = c_0 + \sum_{j=1}^k a_j \cos\left(\frac{2j\pi t}{T}\right) + \sum_{j=1}^k d_j \sin\left(\frac{2j\pi t}{T}\right)$$

## Determination of harmonic number

Visualisation of inverse Fourier transformation

7 harmonics were enough, but 11 kept for analyses

Q: How many coefficients?



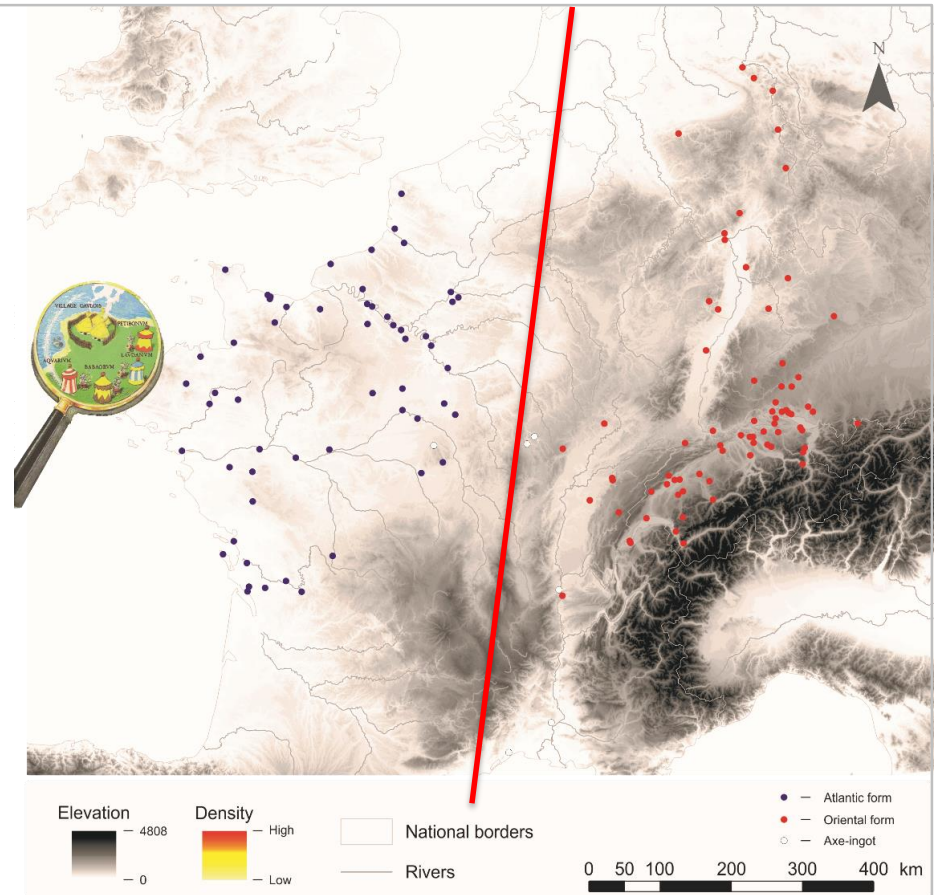
## Checking the current typology (Oriental vs Atlantic group)

Principal Component Analysis  
(PCA)

Self Organising Maps (SOM)

Multidimensional Analysis  
of Variance (MANOVA)

Linear Discriminant Analysis  
(LDA)



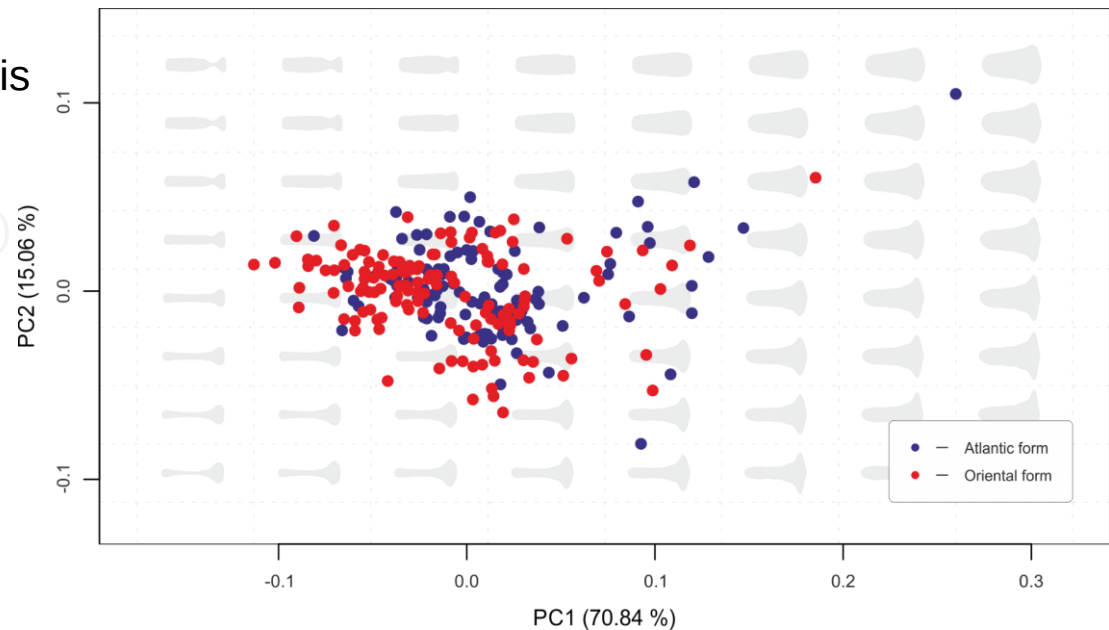
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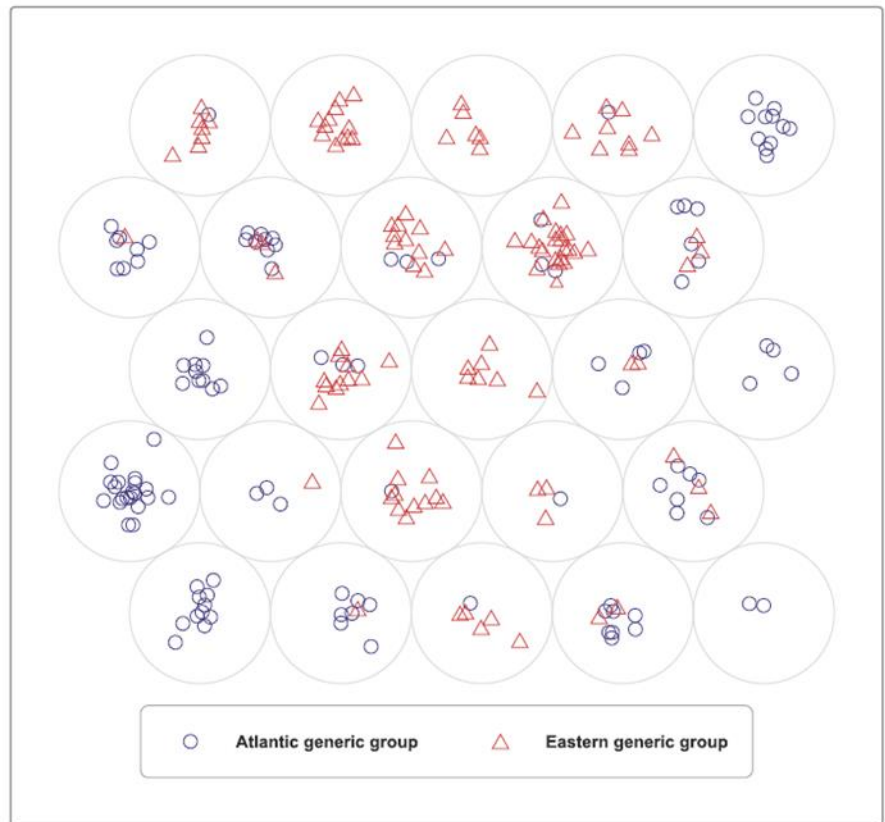
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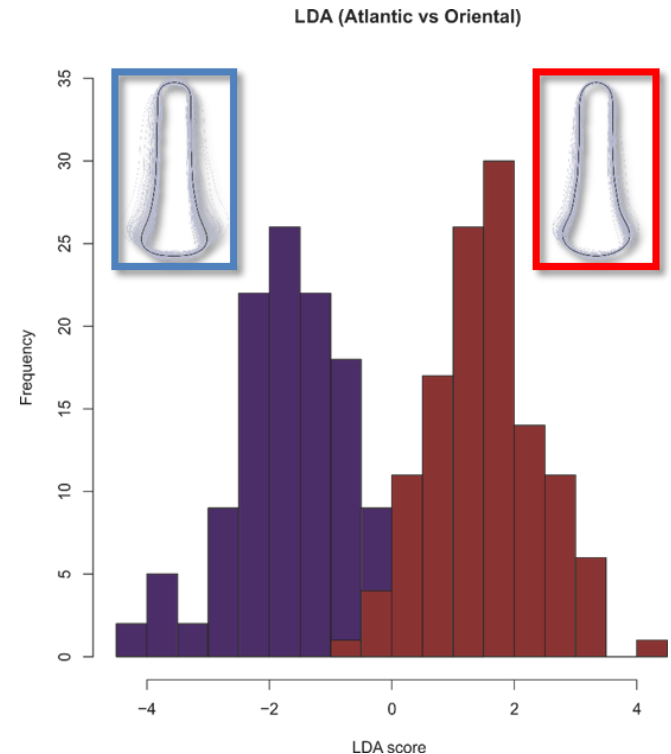
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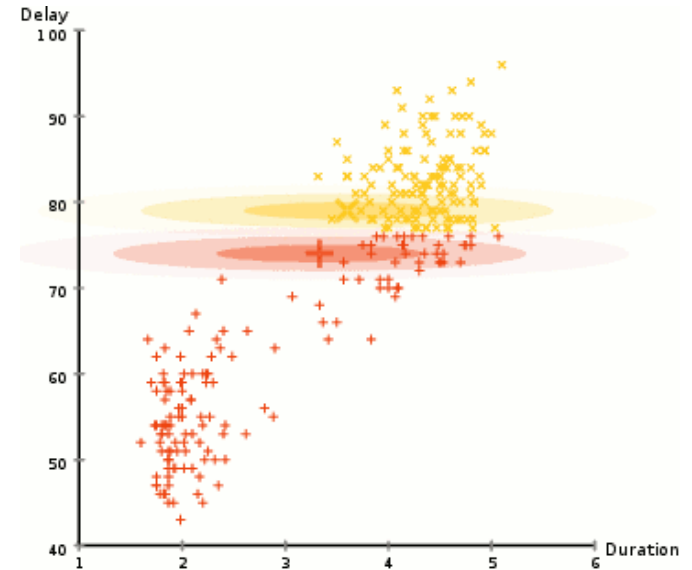
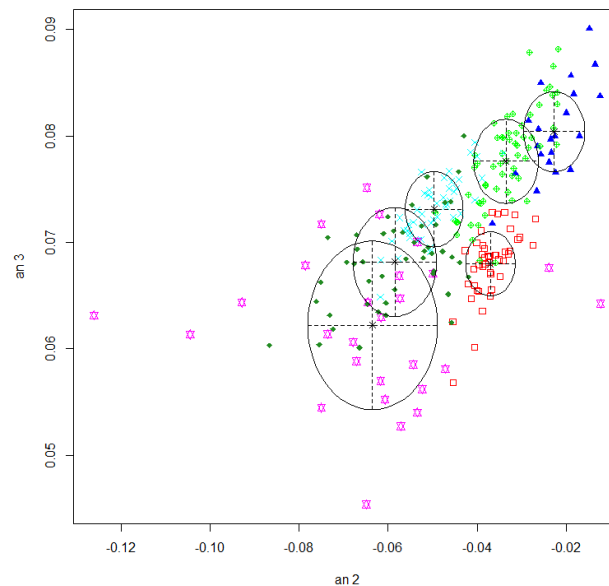
      Df Hotelling-Lawley approx F num Df den Df    Pr(>F)
GR2.groups  1          2.3211   11.605   41   205 < 2.2e-16 ***
Residuals 245
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
  
```



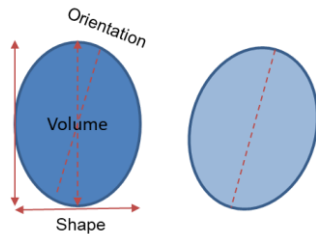
85% success (leave-one-out)  
88% success (cross-validation)

## Making new classification

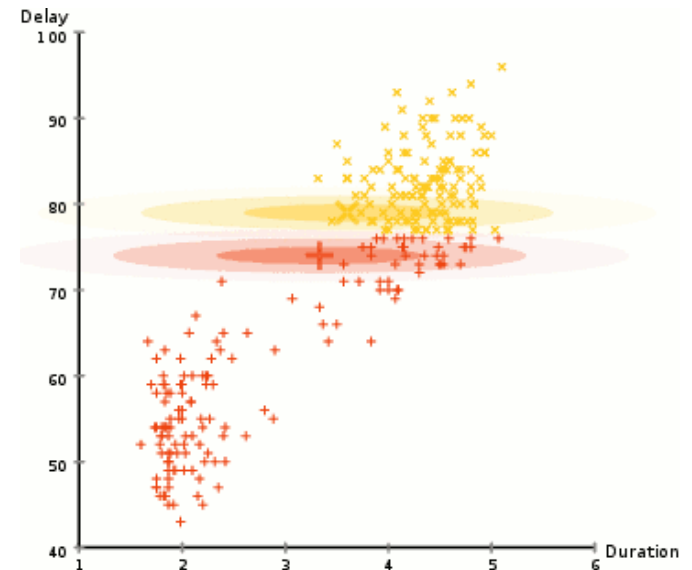
### Gaussian Mixture Model clustering (GMM)



## Gaussian Mixture Model clustering (GMM)



| Model | Volume             | Shape           | Orientation     |
|-------|--------------------|-----------------|-----------------|
| EII   | equality           | identity        | identity        |
| VII   | variability        | identity        | identity        |
| EEI   | equality           | equality        | identity        |
| VEI   | <b>variability</b> | <b>equality</b> | <b>identity</b> |
| EVI   | equality           | variability     | identity        |
| VVI   | variability        | variability     | identity        |
| EEE   | equality           | equality        | equality        |
| EEV   | equality           | equality        | variability     |
| VEV   | variability        | equality        | variability     |
| VVV   | variability        | variability     | variability     |



## Checking new classification

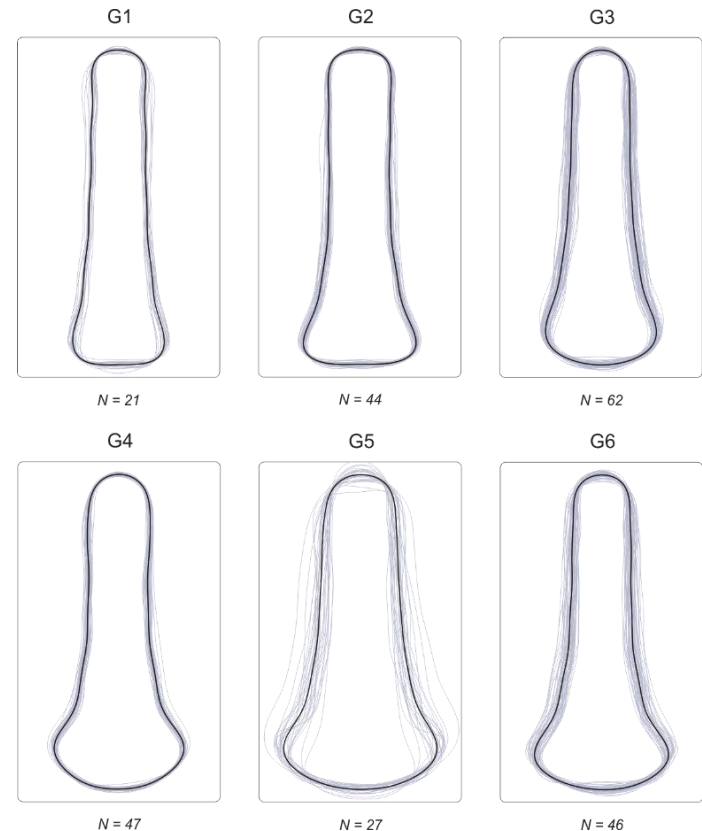
Visual observation

Self Organising Maps (SOM)

Linear Discriminant Analysis (LDA)

Kernel density

Multinomial Scan Statistic (SatScan)



## Checking new classification

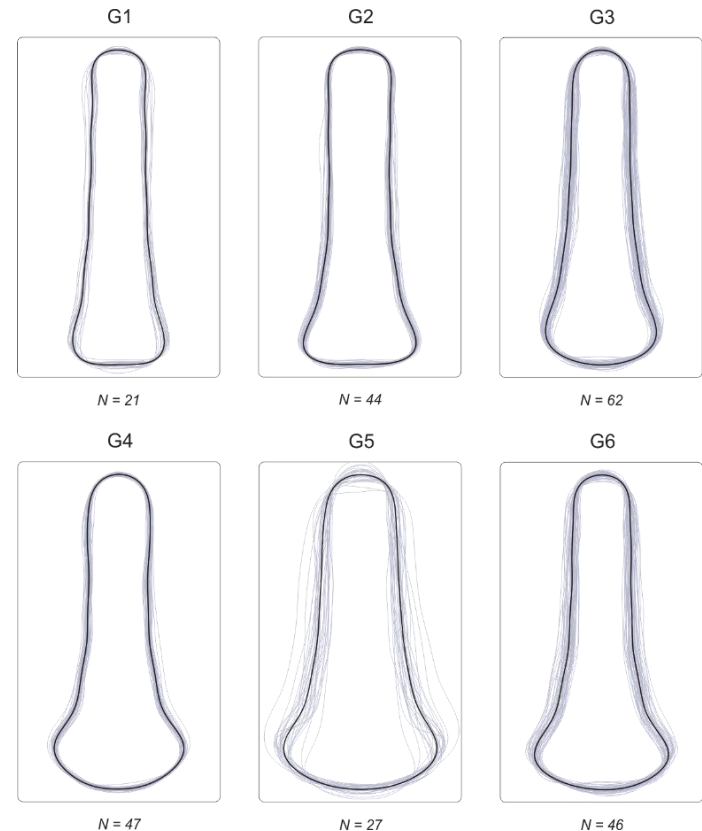
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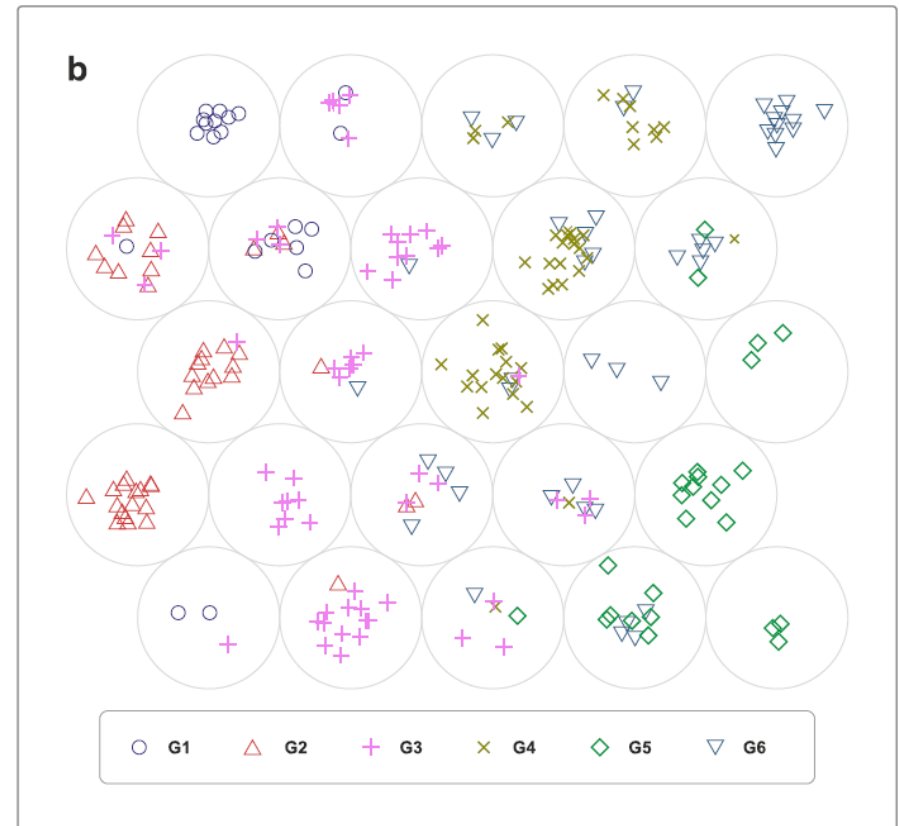
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Visual observation

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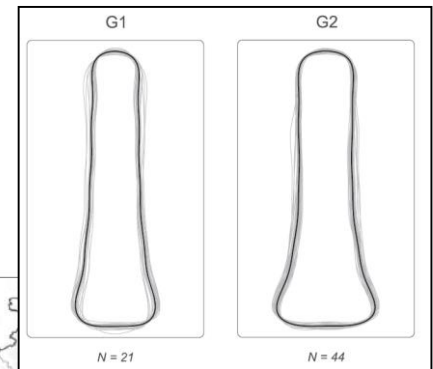
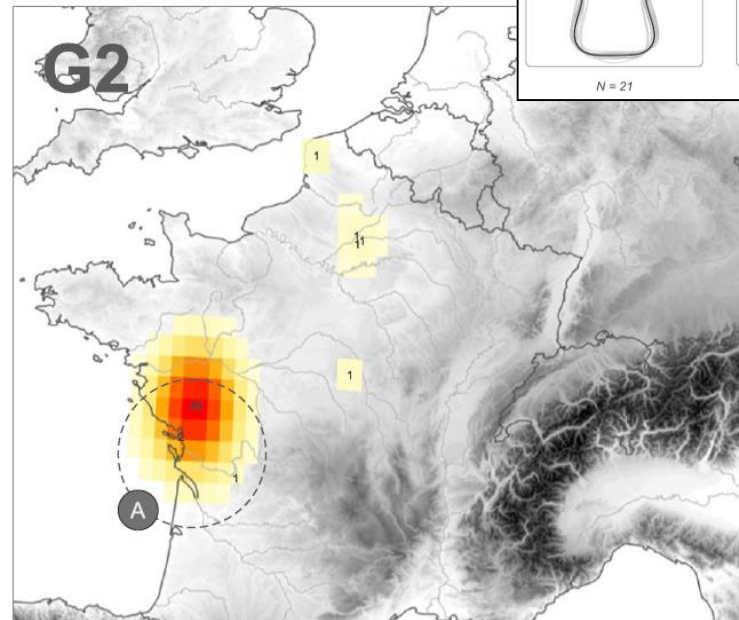
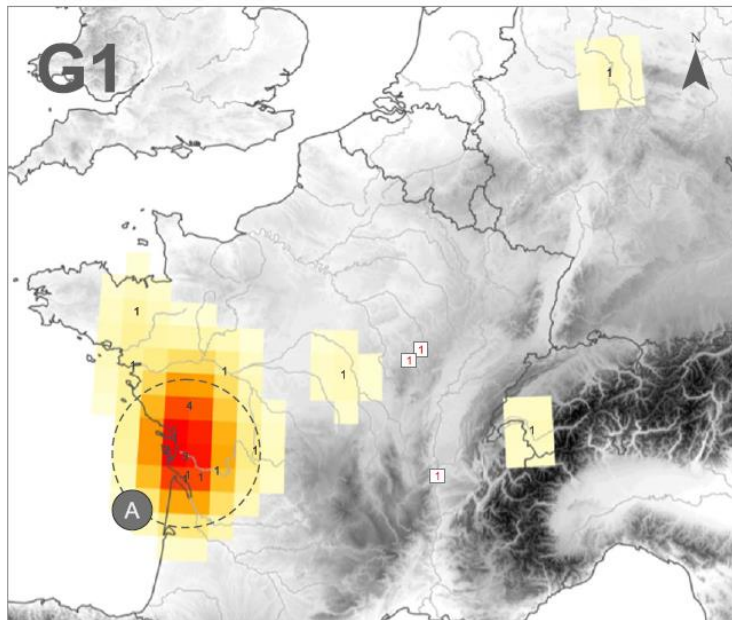
Kernel density

Multinomial Scan Statistic (SatScan)

none of the specimens was  
erroneously  
classified

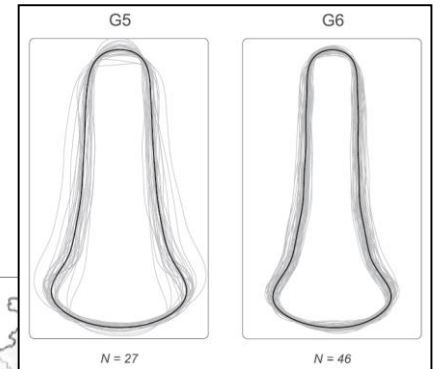
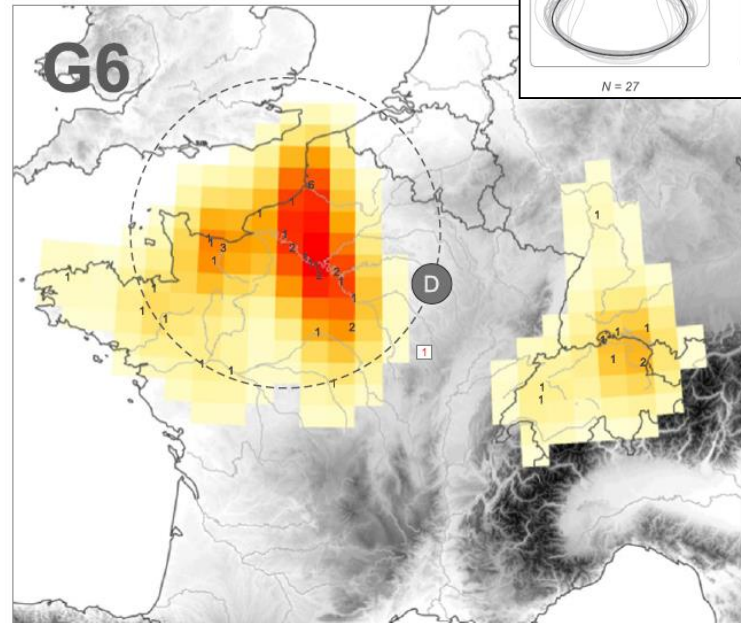
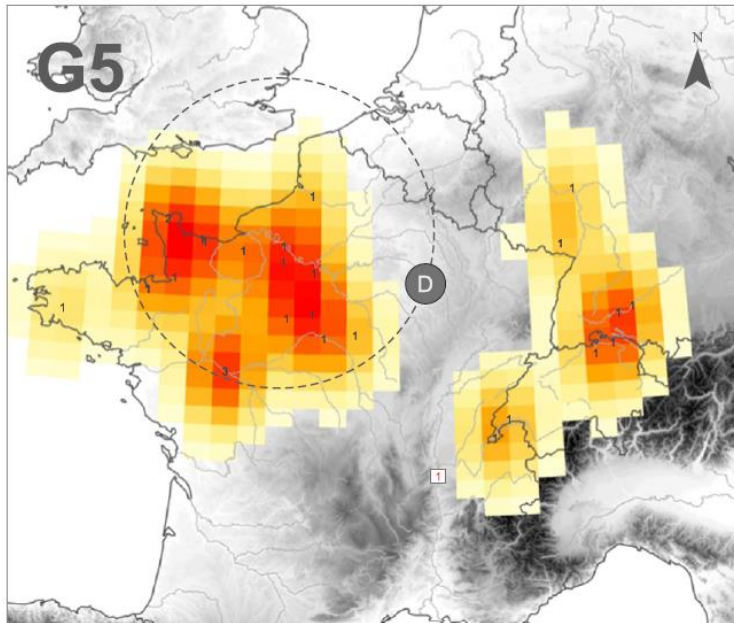
## Checking new classification

Kernel density and Multinomial Scan Statistic (SatScan)



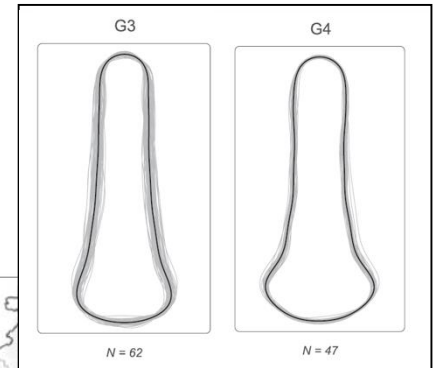
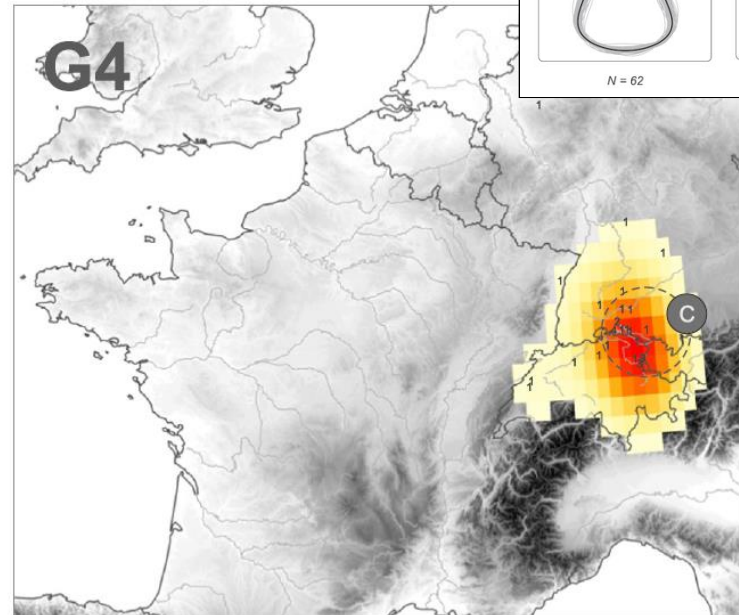
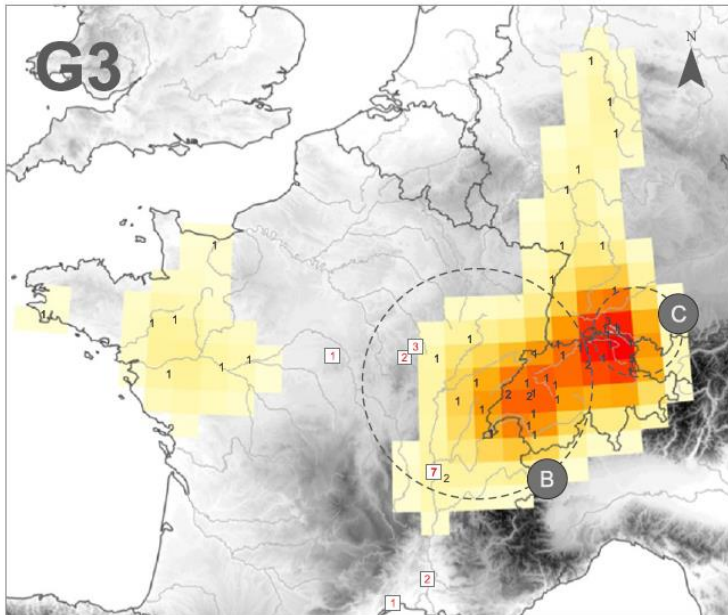
## Checking new classification

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## Checking new classification

Kernel density and Multinomial Scan Statistic (SatScan)

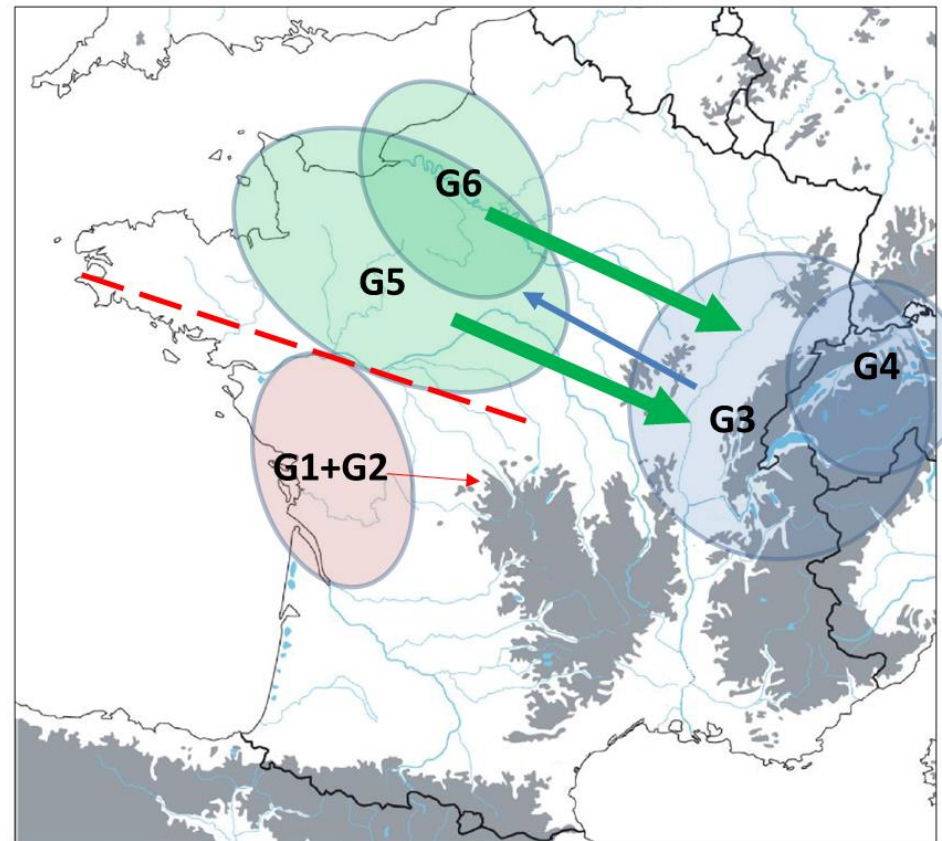
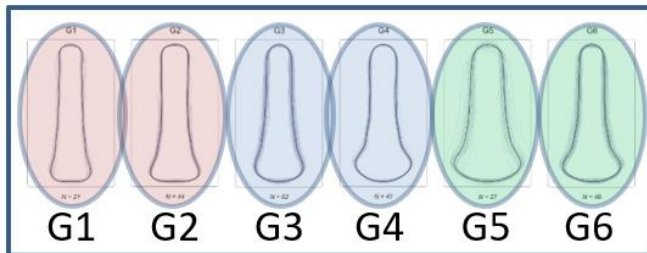


## Interpretations...

Exchanges:

|                               |     |
|-------------------------------|-----|
| west $\Leftrightarrow$ east   | yes |
| south $\Leftrightarrow$ north | no  |

**Economic/cultural “blockade”**



## References:

- Forel, B., 2009. La métallurgie des alliages à base de cuivre en Bourgogne et en France orientale à l'âge du Bronze. Approche paléoenvironnementale et étude du mobilier métallique. Université de Bourgogne, Dijon.
- Karasik, A., Smilansky, U., 2011. Computerized morphological classification of ceramics. J. Archaeol. Sci. 38, 2644–2657.
- Králík, M., Urbanová, P., Hyršovská, J., 2008. Hodnocení stop na stavební keramice ze středověké cihelny v Sezimově Ústí, in: Krajíc, R. (Ed.), Středověké Cihlářství. Sezimovo Ústí Archeologie Středověkého Poddanského Města. Jihočeská univerzita v Českých Budějovicích & Husitské muzeum v Táboře, České Budějovice & Tábor, pp. 207–276.
- Lestrel, P.E., 1989. Method for Analyzing Complex Two-Dimensional Forms: Elliptical Fourier Functions. Am. J. Hum. Biol. 1, 149–164.
- MacLeod, N., 1999. Generalizing and extending the eigenshape method of shape space visualization and analysis. Paleobiology 25, 107–138.
- Monna, F., Jebrane, A., Gabillot, M., Laffont, R., Specht, M., Bohard, B., Camizuli, E., Petit, C., Chateau, C., Alibert, P., 2013. Morphometry of Middle Bronze Age palstaves. Part II - spatial distribution of shapes in two typological groups, implications for production and exportation. J. Archaeol. Sci. 40, 507–516.
- Saragusti, I., Karasik, A., Sharon, I., Smilansky, U., 2005. Quantitative analysis of shape attributes based on contours and section profiles in artifact analysis. J. Archaeol. Sci. 32, 841–853.
- Urbanová, P., 2009. A Study of Human Craniofacial Variation by Using Geometric Morphometrics. Masaryk university, Brno.
- Wilczek, J., 2017. New approaches for the acquisition, systematisation and interpretation of archaeological artefacts. Masarykova univerzita & Université Bourgogne - Franche-Comté, Brno & Dijon.
- Wilczek, J., Monna, F., Gabillot, M., Navarro, N., Rusch, L., Chateau, C., 2015. Unsupervised model-based clustering for typological classification of Middle Bronze Age flanged axes. J. Archaeol. Sci. - Rep. 3, 381–391.
- <https://jwilczek.com/teaching/m2-ages-morphometrie/>
- <http://www.anarchiel.com>
- <http://www.biology-online.org>
- <http://www.fabricemonna.com/enseignement-2/master-ages/morphometrie-m2-ages/>
- <http://www.wikipedia.com>