

Computer-assisted Orientation and Drawing of Archaeological Pottery

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Archaeologists spend considerable time orienting and drawing ceramic fragments by hand for documentation, to infer their manufacture, the nature of the discovery site and its chronology, and to develop hypotheses about commercial and cultural exchanges, social organisation, resource exploitation, and taphonomic processes. This study presents a survey of existing solutions to the time-consuming problem of orienting and drawing pottery fragments. Orientation is based on the 3D geometry of pottery models, which can now be acquired in minutes with low-cost 3D scanners. Several methods are presented: they are based on normal vectors, or circle fittings, or profile fittings. All these methods seek to determine the optimal position of the rotation axis. We also present and discuss new approaches and improvements to existing methods. We have developed a suite of functions for the computer-assisted orientation and drawing of archaeological pottery. The profile and contours of the fragment, as well as any possible decoration, can be depicted in various ways: photorealistic rendering or dotted patterns, calculated by ambient occlusion, combined or not with artificial light. The general workflow, evaluated using both synthetic and real-world fragments, is rapid, accurate, and reproducible. It drastically reduces the amount of routine work required to document ceramic artefacts. The information produced, together with the 3D representation of the fragments, can easily be archived and/or exchanged within the archaeological community for further research. The source code (built in the R environment), together with an installation notice and examples, is freely downloadable.

CCS Concepts: • **Applied computing** → **Archaeology**;

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XXXX-XXXX/2018/1-ART1 \$15.00

<https://doi.org/10.1145/3230672>

Additional Key Words and Phrases: Archaeology, Documentation, (Semi-)automatic pottery orientation, Pottery illustration, 3D reconstruction

ACM Reference Format:

Josef Wilczek, Fabrice Monna, Ahmed Jébrane, Catherine Labrère Chazal, Nicolas Navarro, Sébastien Couette, and Carmela Chateau Smith. 2018. Computer-assisted Orientation and Drawing of Archaeological Pottery. *ACM J. Comput. Cult. Herit.* 1, 1, Article 1 (January 2018), 19 pages. <https://doi.org/10.1145/3230672>

1 INTRODUCTION

Pottery fragments are among the most abundant artefacts found during archaeological surveys and excavations. Together with their function, manufacture and chronology, they provide evidence of human organisation, resource exploitation and taphonomic processes [46, 51]. Recent developments in 3D computer graphics and pattern recognition have led to prominent advances in the treatment of archaeological pottery, aiming to provide semantic data-based systems, allowing the annotation, retrieval, and reassembly of pottery fragments, and predictive digitisation of vessels [8, 19, 38, 47–49, 68]. However, despite of these advances, a practical and easy-to-use solution to the time-consuming problem of methodical archaeological illustration is still not readily available for the large-scale documentation of archaeological remains. Current archaeological illustrations are standardised, at least in part (see e.g. [1, 9, 16, 18, 58, 60]). Archaeological pottery drawings are generally divided into two parts: the central line represents the axis of rotation, the left part depicts the vertical section with any inner details; the right part depicts the outer section. The position of the rotation axis, which is of primary importance for further classification, is traditionally defined by positioning the rim or base of the fragment on a radius chart. Once the fragment has been oriented and its diameter determined, the profile outline is obtained by a profile gauge, drawn manually in a cutaway view, then scanned and processed with graphical software to produce the final raster or vector image. Drawing pottery is thus a time-consuming and costly task, especially when dealing with hundreds or even thousands of fragments. Since the 1990s, numerous articles have been published on how to automate the orientation process (e.g. [34, 50]). Most methods exploit three fundamental geometric properties of rotationally symmetrical objects (Fig. 1): (i) normal vectors of the surface pass through the axis of rotation (Fig. 1:a; e.g. [4, 20–23]); (ii) horizontal planes intersecting the object form circles whose centres lie on the axis (Fig. 1:b; e.g. [22, 32, 39, 40, 42]); and (iii) fragment profiles projected to the same vertical plane occupy the same location (Fig. 1:c; e.g. [33, 34]). Note that the first two properties do not concern surfaces that are orthogonal to the axis, such as flat bases. Our goal here is therefore to inventory existing approaches for rotation axis estimation, based on normal vectors, horizontal/vertical sections, and to introduce new methods capable of segmenting fragments (outer and inner surfaces), and discarding parts that do not provide information about the rotation axis (fractures, plastic decoration, etc.). Most published studies have focused on theoretical aspects. The description of the algorithms used is often succinct and the code is not always available, so that it is difficult to replicate the procedures involved, for quantitative comparison. Rotation axis estimation has often been tested on only a few large, regular, well-preserved fragments. The final illustrations are frequently schematic, and do not necessarily integrate all the information about the fragment surface. Some authors have proposed practical tools or, at least, pipelines (e.g. [13, 52, 53]), but no multiplatform solution has so far been made available to the archaeological community, except to some extent the Gigamesh software (<http://www.gigamesh.eu> [41]), or intra-laboratory software, available upon request [34]. Note that these solutions are not yet complete; they do not cover all existing procedures, and/or do not always allow archaeological illustrations to be produced in conformity with current standards. We have developed approaches based on these existing state-of-the-art methods, adopting optimal procedures for each part of the process. The material and methods section therefore provides a detailed description of these proposed approaches. A fully automatized workflow (i.e. the most constraining) is tested on datasets consisting of both synthetic models and real-world archaeological fragments. Results are evaluated

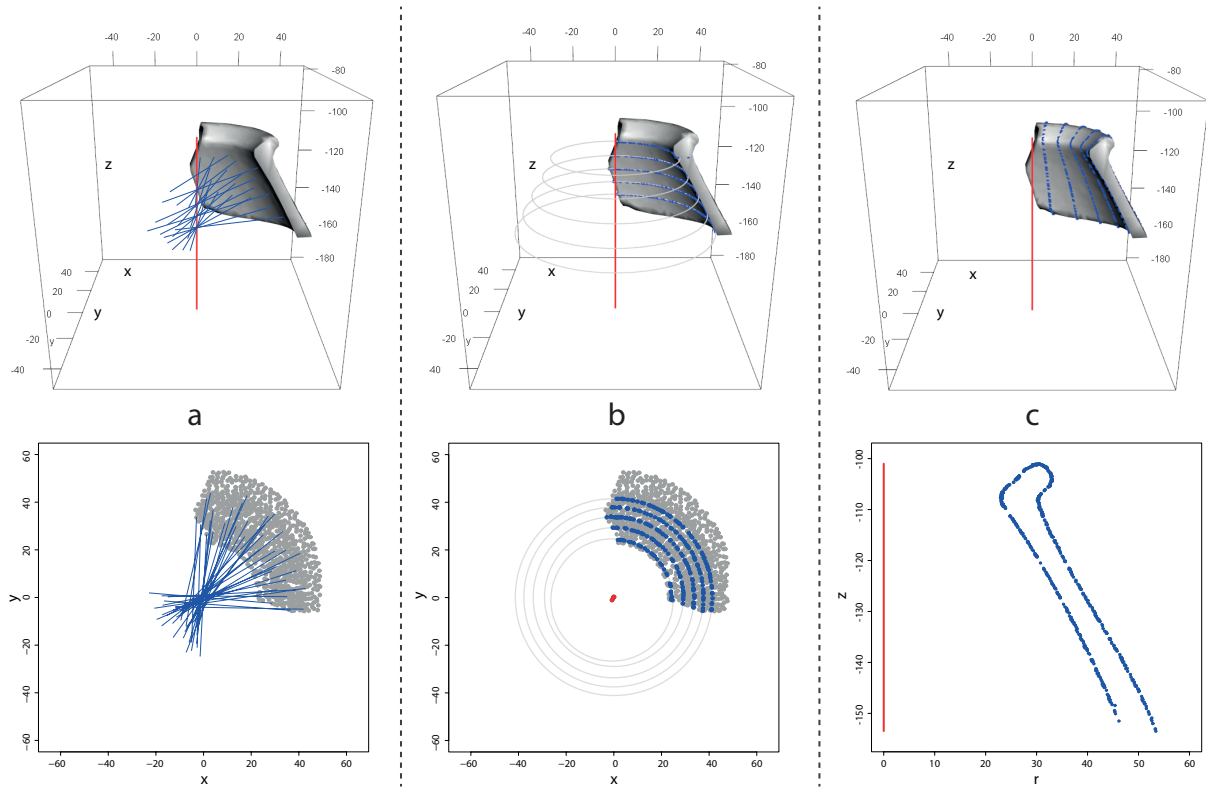


Fig. 1. Three main geometric properties of the rotationally symmetrical objects exploited for rotation axis estimation. a) Normal vectors of vertices lying on the pottery surface pass through the axis of rotation. b) Horizontal planes intersecting a fragment form circles whose centres lie on the rotation axis. c) Fragment profiles projected to one plane occupy the same location.

in terms of reproducibility and then compared with traditional illustrations produced by specialists. The new tools presented here for pottery orientation are freely available, as a suite of functions encoded in R software [62], including the production of archaeological illustrations, adapted to most norms and standards of pottery drawings (linear and shaded drawing, photographic representation, etc.). A user-friendly interface is proposed, with many options and tuneable parameters, to address problems of illustration likely to be encountered by the archaeological community.

2 MATERIAL AND METHODS

2.1 Acquisition

The 3D model of the pottery fragment can be obtained by any method (CT scan, photogrammetry, laser scan, etc.), but a 3D scanning system based on structured light is a suitable choice at this step because of its rapidity, adequate accuracy, and simplified manipulation. For the present study, 110 pottery fragments (rim, body and base), different in size, shape, and/or state of preservation, from 3 chronological periods (Neolithic, Bronze Age, and

Roman), produced using various fabrication techniques, were scanned with an EinScan-S structured light-based 3D scanner (resolution <100 μm). The entire acquisition process, including scanning and post-processing, took at most 5 minutes per fragment. Typically, the number of faces used for further calculation was limited to 500,000, which will allow further drawing of complex decoration, while keeping computation time to a minimum. In any case, possible defects of the model, such as duplicated or unreferenced vertices, or non-manifold faces, should be repaired prior to treatment. The synthetic data used to test the quality of automatic orientation was produced as follows. A 3D pottery model was created in the free Blender software (<https://www.blender.org/>) by rotating the vectorised profile of a 30-cm-high vessel from the Bibracte *oppidum* (Burgundy, France) around its axis of rotation. As the resulting model was perfectly symmetrical, deformation and noise were applied to the surface, to better simulate real-life conditions. The reconstructed vessel was randomly broken into 34 pieces, using a Voronoi fracture diagram. All fragments were then rotated, translated and shuffled in the 3D space, as the starting point of the experiment.

2.2 Model Pre-Orientation

A fragment can be pre-oriented either manually or automatically. As difficult-to-process 3D models usually contain a large number of vertices, Quadric Edge Collapse Decimation [15] was used to reduce the number of vertices to approximately 10,000. To lessen the influence of features containing no information about the rotation axis (e.g. decoration), we opted to smooth the model, using a Laplacian Smoothing algorithm [63].

2.2.1 Manual Pre-Orientation. Manual pre-orientation exploits the fact that an expert is often intuitively able to pre-orient the fragment simply by perceiving its overall shape, or by identifying some specific horizontal features (e.g. rim, base, wheel-thrown lines on the inner side of fragment, horizontal decoration, etc.). Three points can then be defined within the same horizontal plane. The centre of the circle passing through these three points is determined using least-squares fitting [6]. The model is shifted and rotated to make the rotation axis coincide with the z-axis of the orthonormal coordinate system (Fig. 2). All parts with normals that do not point to the rotation axis (i.e. fracture surfaces, handles, plastic decoration, base, and rim), must be removed. Pertinent surfaces are then extracted by first calculating the distance, $d(n_i, Z)$, between the straight line n_i defined by each vertex X_i and its corresponding normal vector \vec{N}_i , and the rotational axis Z defined by $X_0 = [0, 0, 0]$ and $\vec{N}_0 = [0, 0, 1]$ (Equation (1); [20, 21]):

$$d(n_i, Z) = \frac{(X_i - X_0) \cdot (\vec{N}_i \times \vec{N}_0)}{\|\vec{N}_i \times \vec{N}_0\|} \quad (1)$$

In our approach, only the points presenting the lowest distances (e.g. 50%) were retained, as these points contain sufficient information for pre-alignment. It is then fairly straightforward to separate the inner and outer surfaces of the model by examining whether a vertex shifted by its normal is closer to the rotation axis or not.

2.2.2 Automatic Pre-Orientation. Automatic model pre-orientation in our approach consists of two steps: (i) elimination of vertices that are not part of the surface of revolution; and (ii) estimation of the rotation axis by normal vectors, using robust regression. Vertices located on non-regular features (i.e. fracture surfaces or plastic decoration) do not belong to the surface of revolution, and must therefore be discarded. As opposing vessel surfaces are approximately parallel, the normal vectors (e.g. \vec{N}_i and \vec{N}'_i) of two vertices (e.g. X_i and X'_i) lying exactly opposite each other should point in opposite directions (Fig. 3:a). The probability, p , that any two normal vectors point in opposite directions was evaluated following Li [37] and Schlager [56, 57], and only those vertices, X_i , where the vectors have a p value higher than a defined threshold value (e.g. 0.95) were retained. After eliminating vertices that are not part of the surface of revolution, the automatic pre-orientation process then seeks a line (i.e. the axis of rotation) for which the sum of the squared distances to normals is minimal (Fig. 1:a).

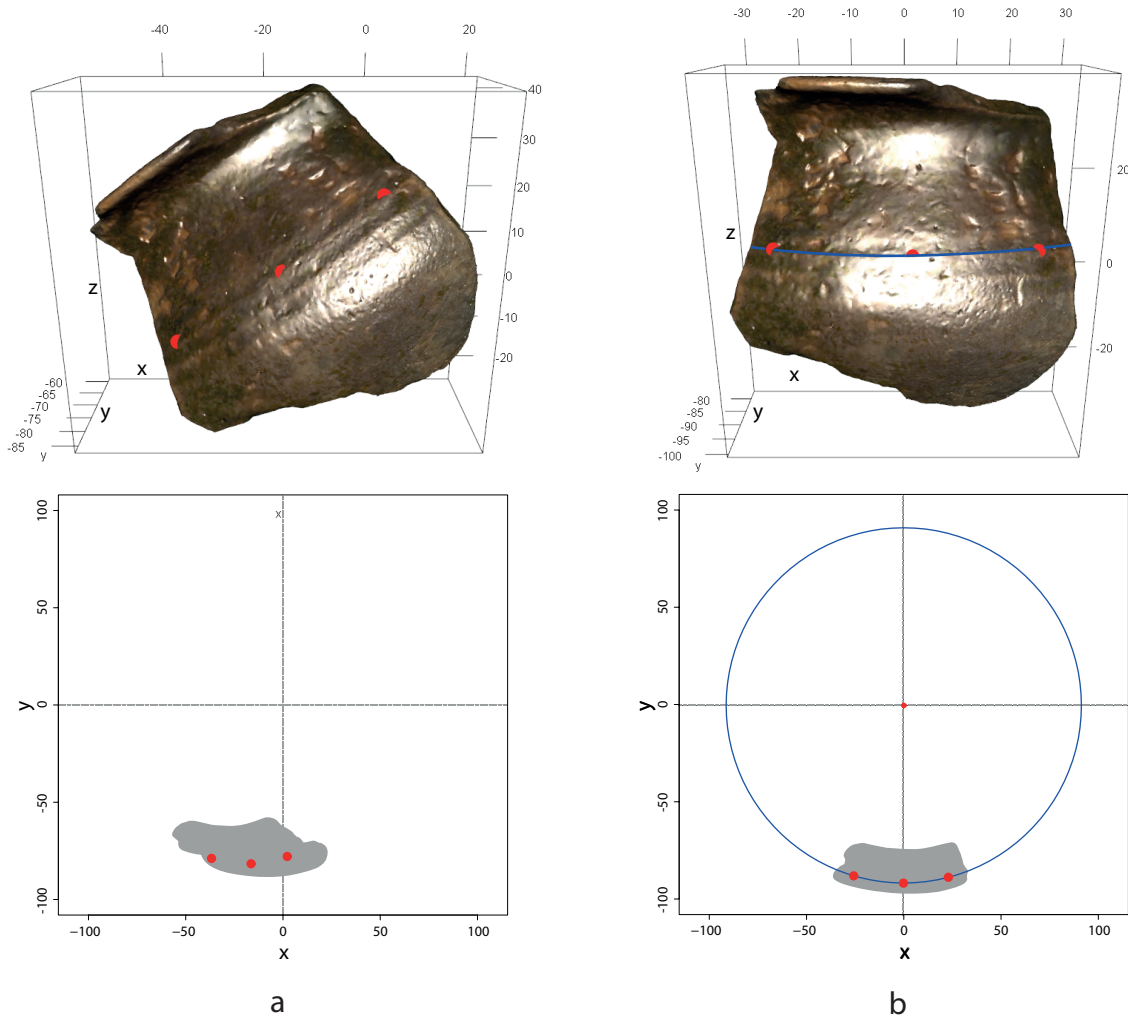


Fig. 2. Manual pre-orientation. a) Three points (red dots) defined on the model. b) The model is rotated by the plane defined by the three points, and a circle (blue line) is then fitted to the three points. The model is then shifted along the x- and y-axes to coincide with the rotation axis.

Two approaches exist for this process: the Hough Transform, by tracing normals through the accumulator space (e.g. [23, 27, 28, 66]), and direct least-squares optimisation (e.g. [20–22]). This second approach has often been thought to lack robustness (e.g. [42]), but a modified version was successfully applied here. As demonstrated by Halíř [20, 21], the position of the vertex (X_0) on the estimated axis in Equation (1) is linearly related to the normal vector direction of the axis (\vec{N}_0). The search for the axis of rotation can therefore be simplified as a two-parameter optimisation problem (calculated here by the Nelder-Mead algorithm [44, 45]): φ for rotation around the x-axis and θ for rotation around the y-axis (see [20, 21] for more details). To improve the robustness of Halíř's approach,

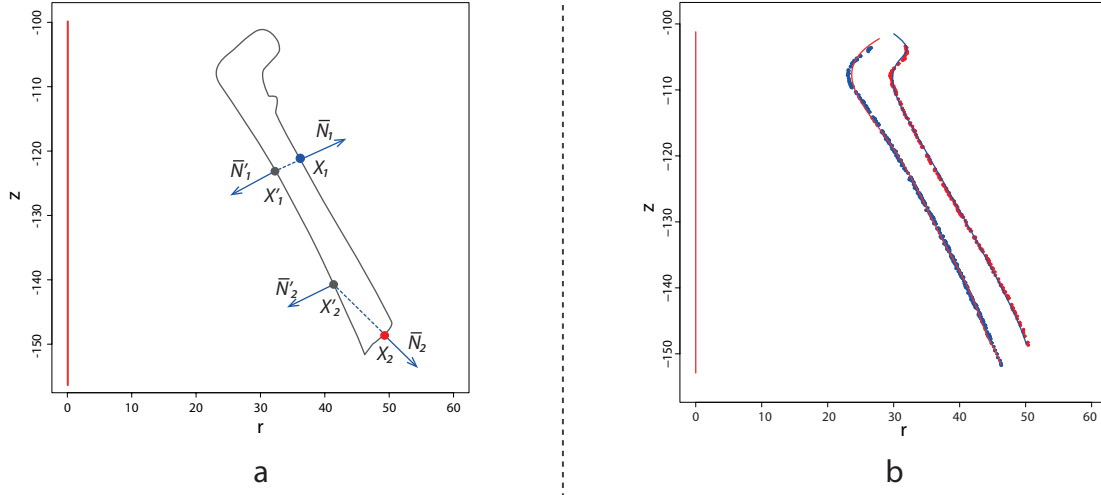


Fig. 3. Extraction of surfaces relevant to axis estimation (a) and vertical profile superimposition using polynomials (b). a) Vertex lying on parallel (blue dot) and non-parallel (red dot) fragment surfaces. b) Fragment vertices of inner (blue dots) and outer (red dots) surfaces are perfectly superimposed and adjusted to the corresponding polynomial (red and blue curves).

the X_0 coordinates were calculated using very robust linear regression [43, 67], after the elimination step. This method is expected to considerably reduce the deleterious impact of outliers among the normal vectors, or any other source of data noise. The model is rotated and translated, once φ and θ have been calculated, so that its axis of rotation coincides with the z -axis. The inner and outer surfaces can then be separated, as described in the manual pre-orientation paragraph above (Section 2.2.1).

2.3 Fine-tuning the Axis of Rotation

Several methods have been proposed to improve the position of the rotation axis. In our approach, the surface choice option (inner, outer, or both) allows the more regular surface to be selected.

2.3.1 Horizontal Circle Adjustment Using the Radius. For an idealised fragment, properly oriented along the z -axis, the horizontal planes intersecting the inner and outer surfaces should form perfect circles (Fig. 1:b). For a real-world object, the distance between all vertices belonging to the same section and the rotation axis, $d(X_{i,j}, Z)$, should thus be as constant as possible. It is therefore a matter of minimising the mean variance of these distances, as follows (Equation (2)):

$$\min_{\varphi, \theta, a, b} \left(\sum_{i=1}^K \frac{1}{k_i} \sum_{j=1}^{k_i} \left[d(X_{i,j}, Z) - \overline{d(X_{i,j}, Z)} \right]^2 \right) \quad (2)$$

where a , b , φ , and θ translate and rotate the model to make it coincide with the rotation axis, K is the number of horizontal sections, and k_i the number of points in the i -th horizontal section. The minimisation function here requires the optimisation of four parameters (φ , θ , a and b). However, the search process can be performed with only two parameters (a and b), to keep the three points set for manual pre-orientation on the same horizontal plane.

2.3.2 Horizontal Circle Adjustment Using the Multi-Criteria Approach. In addition to the property stated in section 2.3.1., the centres of circles previously calculated and projected on to the xy-plane should all be superimposed on the rotation axis (Fig. 1:b; e.g. [21, 22, 32, 34, 42, 54]). These two criteria can be treated simultaneously, using a multi-objective optimisation procedure (also known as Pareto). However, as the calculation of the circle centres by least-squares fitting is sensitive to the sector angle, in our approach, its value was used to weight both cost functions (note that a minimum number of points and minimum sector angle can be specified in advance). The first (objective) cost function minimises the variability of the circle centres projected on to the xy-plane (Equation (3)):

$$\min_{\varphi, \theta} \left(\frac{1}{\sum_{i=1}^K w_i} \sum_{i=1}^K w_i d(C_i, \bar{C})^2 \right) \quad (3)$$

where C_i is the centre of the optimal circle obtained by regression from points belonging to the i -th horizontal section, and \bar{C} gives the mean coordinates of these centres. The weight, w_i , is the value of the sector angle expressed in radians. The second cost function minimises the mean squared distances between each point and its corresponding circle centre (Equation (4)):

$$\min_{\varphi, \theta} \left(\frac{1}{\sum_{i=1}^K w_i} \sum_{i=1}^K \frac{w_i}{k_i} \sum_{j=1}^{k_i} \left[d(X_{i,j}, C_i) - R_i \right]^2 \right) \quad (4)$$

where R_i is the radius of the regression circle. With this approach, the translation parameters a and b no longer integrate the optimisation process. Once φ and θ have been determined, a and b are straightforwardly drawn from the mean circle centre (\bar{C}). As the model is already pre-oriented, the search process for φ and θ can be limited (e.g. ± 10 degrees), rather than searching within the entire solution space. The Pareto front represents a set of optimal non-dominated solutions in a simple two-dimensional graph, where both x- and y-axes correspond to both objectives. From the visual inspection of this graph, it is possible to decide which solution (or average of a set of solutions) is the most convenient [11], or to select one objective rather than the other, depending on the fragment.

2.3.3 Vertical Profile Superposition. The profiles corresponding to the intersection of the vertical planes passing through the rotational axis and the fragment, projected on to the rz-plane (where r is the radius) should be superimposed (Fig. 1:c; e.g. [29, 33, 34]). In our approach, the longest profile was first set as reference, and the optimal position was obtained when the sum of squared distances between the vertices of the reference profile and those of the other profiles was minimal (Equation (5)):

$$\min_{\varphi, \theta, a, b} \sum_{i=1}^M \left(\min_{\rho_j, \zeta_j} \left((r_i - \rho_j)^2 + (z_i - \zeta_j)^2 \right) \right) \quad (5)$$

where M is the total number of vertices, (r_i, z_i) the rz-coordinates of the points, and (ρ_j, ζ_j) the rz-coordinates of the reference section. Note that this method can be applied to the entire profile. The inner and outer reference profiles can also be considered as polynomial functions. In an ideal case, all vertices projected on to the same rz-plane should perfectly fit these curves (Fig. 3:b; [64, 65]). The optimal position of the model is found by minimising the sum of squared residuals between vertices and reference polynomial functions, $f(z_i)$ as in Equation (6).

$$\min_{\varphi, \theta, a, b} \sum_{i=1}^M \left(r_i - f(z_i) \right)^2 \quad (6)$$

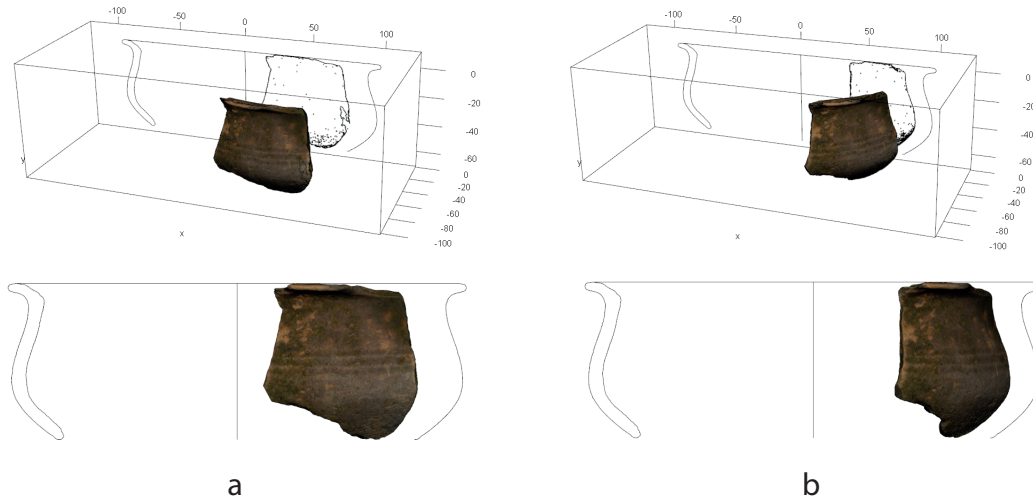


Fig. 4. Different options for fragment orientation used in illustration.

2.3.4 Tangent Plane to Rim and Base. This method uses the traditional principles by which the specialist orients fragments: the optimal orientation is found when the rim/base vertices coincide with the horizontal plane, and the axis of rotation is a line perpendicular to the centre of the circle passing through these vertices [34]. The rim (base) vertices are selected as those presenting the highest (lowest) z -values within each vertical section, and their distance to a horizontal plane is minimised (Equation (7)):

$$\min_{\varphi, \theta, a, b} \frac{1}{\sum \delta_j} \sum_{j=1}^L \delta_j (z_j - \max z)^2 \quad (7)$$

where L is the number of vertical sections and z_j is the z -coordinate of the point (for base fragments, $\min z$ is used instead of $\max z$). The number δ_j is equal to 1 if $|z_j - \max z| < \epsilon$ (e.g. 3 mm), or equal to 0 to eliminate inappropriate points (e.g. when the rim or base of the fragment is damaged).

2.3.5 Oriented Model Exportation. The final oriented model, produced using any of the above approaches, can be exported in Polygon File Format (PLY), and can be used for archaeological illustration, or 3D reconstruction.

2.4 Archaeological Illustration

2.4.1 General Layout. Layouts are represented in the xz -plane, with the rotation axis coinciding with the z -axis. Fragments are traditionally depicted in front view, to the right of the rotation axis, with no consideration for its real geometric position (Fig. 4:a). Nevertheless, the fragment may be positioned in relation to the axis of rotation (Fig. 4:b), or rotated, for example to depict a specific feature, such as a handle. Note that, at this step, the models were processed in full resolution, using the orientation parameters calculated above from their decimated representation.

2.4.2 Overall Form. The selected profile section is the longest part that contains enough information for further fragment classification (e.g. rim, foot, handles, etc.). Specific features can be included in the drawing, such as scale,

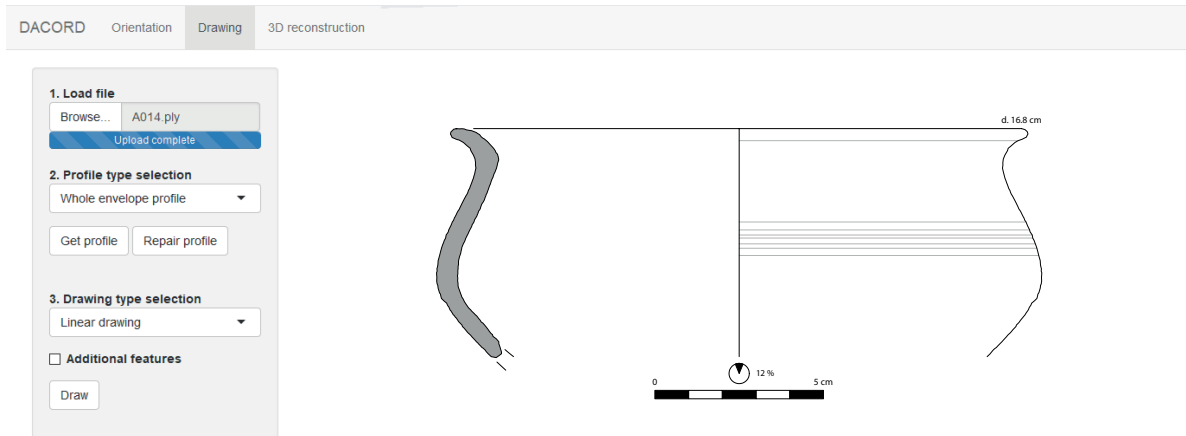


Fig. 5. Example of linear drawing illustration.

distance measurements, volume estimation, maximum sector angle (a proxy for vessel preservation), prolongation of the missing profile parts, and horizontal lines (Fig. 5).

2.4.3 Fragment Rendering. In modern archaeological illustrations, the fragment surface is increasingly represented through colour/greyscale photography. To obtain such illustrations, the colour or greyscale values of the vertices/faces are projected orthogonally on to the xz -plane (Fig. 4; see also [60]). Dotted shading, to give a sense of depth to the drawing, may be used as an alternative. Calculations of the amount of light that a surface receives, and ambient occlusion, which indicates how much ambient light a point receives [25], can be found in most textbooks (e.g. [14]). Ambient occlusion darkens the more occluded points of the 3D model, creating a more realistic appearance [36]. Here, the package ‘shadevis’ written in C++ (<https://sourceforge.net/projects/vcg/files/shadevis/>) and used *via* Meshlab [7] was preferred, because it produces satisfactory results rapidly. Shading was rendered by plotting more dots in the darkest areas (Fig. 6).

2.4.4 Vessel Regularity. Once aligned, possible irregularities in the surface, such as decoration or technological traces, can be revealed by two visualisation methods. The first calculates the distances between model points and the rotation axis, and depicts them on the model surface using a colour code (Fig. 7:a), so that points belonging to the same horizontal section should ideally have the same colour. In the second method, the distance is measured between model points and the corresponding ideal vessel, obtained by rotating a model profile around the rotation axis (Fig. 7:b). A regular model should thus appear as homogeneously coloured as possible (here, in blue corresponding to values close to 0). These representations also illustrate the quality of fragment orientation.

2.4.5 3D Reconstruction of the Preserved Part of the Vessel. Turning one of the profiles along the rotation axis produces a 3D reconstruction of the vessel (Fig. 7:b; e.g. [3, 30, 41]).

2.4.6 Illustration Exportation. All archaeological illustrations produced can be exported in Encapsulated PostScript format (EPS), while oriented models and 3D reconstruction obtained by revolution around the rotational axis can be exported as a triangular mesh (in PLY format).

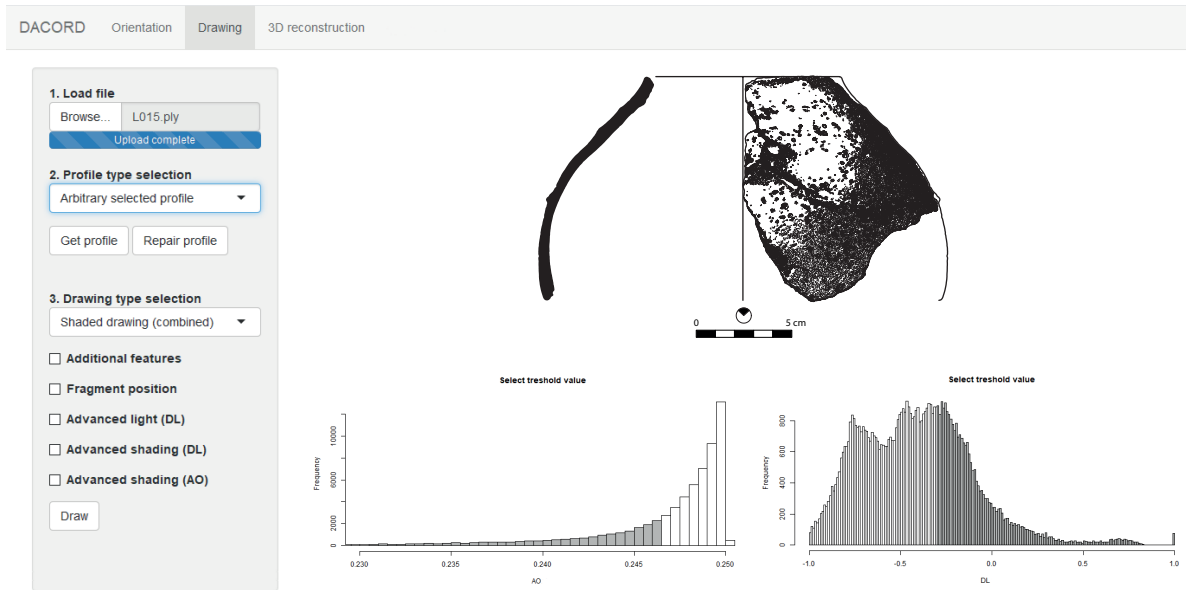


Fig. 6. Example of shaded drawing using a combination of directional lighting and ambient occlusion. Values in distributions used for fragment shading are highlighted in grey.

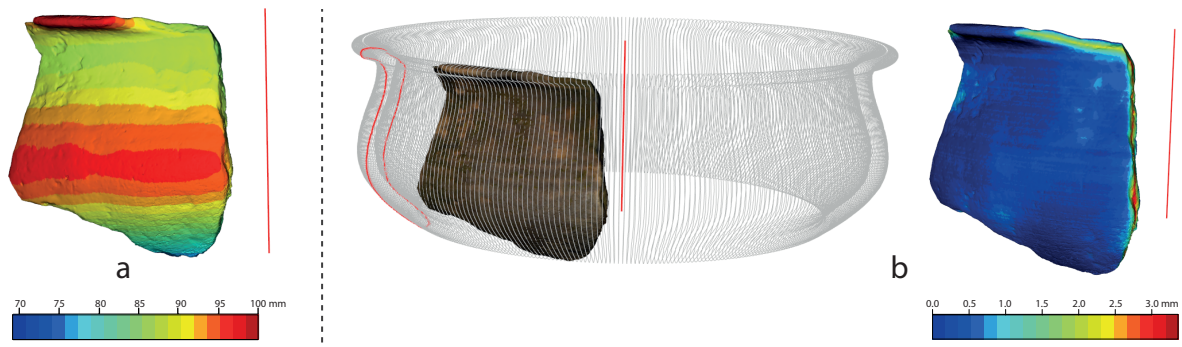


Fig. 7. Two methods of calculating vessel regularity: a) Distances between model points and the axis of rotation; b) Distance between the model (in brown) and the corresponding ideal vessel (in grey), obtained by rotating a given model profile (red polygon), around the axis of rotation.

2.5 Programming and Distribution

The suite of functions written in the R environment, together with instructions and examples, are freely available as Supplementary materials S1. The functions use several packages for calculation, and the Shiny application framework [5], combined with RStudio [61], providing a user-friendly graphical interface, which does not require

any particular knowledge of computer coding. The full list of packages with corresponding references is provided in Supplementary materials S2. It should be noted that all software and packages used are freely available.

3 RESULTS

3.1 Evaluation of the Workflow On a Synthetic Model

A consumer-grade computer (Intel Core i7-2670QM - 2.20 GHz, 8 Go DDR3, NVIDIA GeForce GTX 560M) was used for all the following experiments. Although alignment can be achieved in multiple ways, it was decided to evaluate the quality of the rotation axis position using the least restrictive procedure: the fully automatic pre-orientation via normals, with adjustment refining using the radius of the horizontal circles (performed on both inner and outer model surfaces, with $K = 20$ sections). The time required to process one fragment was less than 3 minutes. Once the rotation axis had been estimated for each fragment (Fig. 8:c), it was then automatically placed as close as possible to its original position, by minimising the mean point-to-point distance between the oriented fragment and the original. Note that, at this step, only translation along the z-axis, and rotation around it, were allowed, so that the quality of the rotation axis position was not affected (Fig. 8:d). Results showed that 29 fragments (85%) were well oriented, with a mean point-to-point distance not exceeding 4 mm (Fig. 8:e, f). This value may appear high, but it should be kept in mind that the original vessel was 30 cm tall, and that it had been severely deformed. The 5 unsatisfactory results concerned extremely small fragments, with flat bases, and/or possessing a sector angle too low to be properly treated. These fragments, which would undoubtedly have been discarded by the archaeologist as untreatable, demonstrate the limits of the fully automated method.

3.2 Evaluation of the Workflow On Real-World Pottery Fragments

All fragments were automatically oriented, as described in the previous paragraph. The distance between each oriented model and its corresponding ideal vessel (Fig. 7:b) was used as a proxy for orientation quality. For the vast majority of fragments ($n = 85$), the axis of rotation was determined almost perfectly, as the distances never exceeded 2 mm (Fig. 9:1-10; for all results see Supplementary materials S3). The normal-based pre-orientation failed for the 25 remaining fragments (Fig. 9:11-20), because they were small (e.g. Fig. 9:11-12), almost flat (e.g. Fig. 9:13-14), they had too great a surface parallel with the rotation axis (e.g. bases or rims; Fig. 9:15-17), or they were almost spherical (e.g. Fig. 9:18-20). However, most of these fragments unsuitable for automatic pre-orientation due their geometry can be processed with rapid manual pre-orientation, simply by placing 3 points on the same horizontal plane (see results with manual pre-orientation in Supplementary materials S3-6). Even for nearly spherical body fragments, the presence of rills may also allow the archaeologist to find a plane perpendicular to the rotation axis, thus making manual pre-orientation possible.

To test the robustness of the alignment procedure, an additional set of 6 rim fragments was processed by 4 archaeologists, using different procedures for pre-orientation and final adjustment. Calculated rim radii were then compared to those obtained by an expert in archaeological illustration (Table 1). The resulting values were very similar whatever the operator and method. The calculated radii differed by less than 13% from those determined by the expert (handmade and computed-made drawings are available in Supplementary materials S4), which are themselves subject to possible errors. Note that over half of the selected fragments had a sector angle of less than 12°, yet even for these difficult cases, satisfactory results were obtained.

As a demonstration of assisted drawing, a set of fragments was represented in linear mode (Fig. 10:1-2), coloured mode (Fig. 10:3-4), shaded modes using directional lighting and/or ambient occlusion (Fig. 10:5-8), and in a mode representing vessel regularity (Fig. 10:9-10). These illustrations possess the qualities required for archaeological documentation. Directional light shading highlights the overall vase relief, and any technological, plastic or curved features (traces, ribs, mouldings, grooves, decoration, etc.). Ambient occlusion reveals local structure, but also fractures, defects, and surface porosity (Fig. 6), independent of the light source position. In less than 5

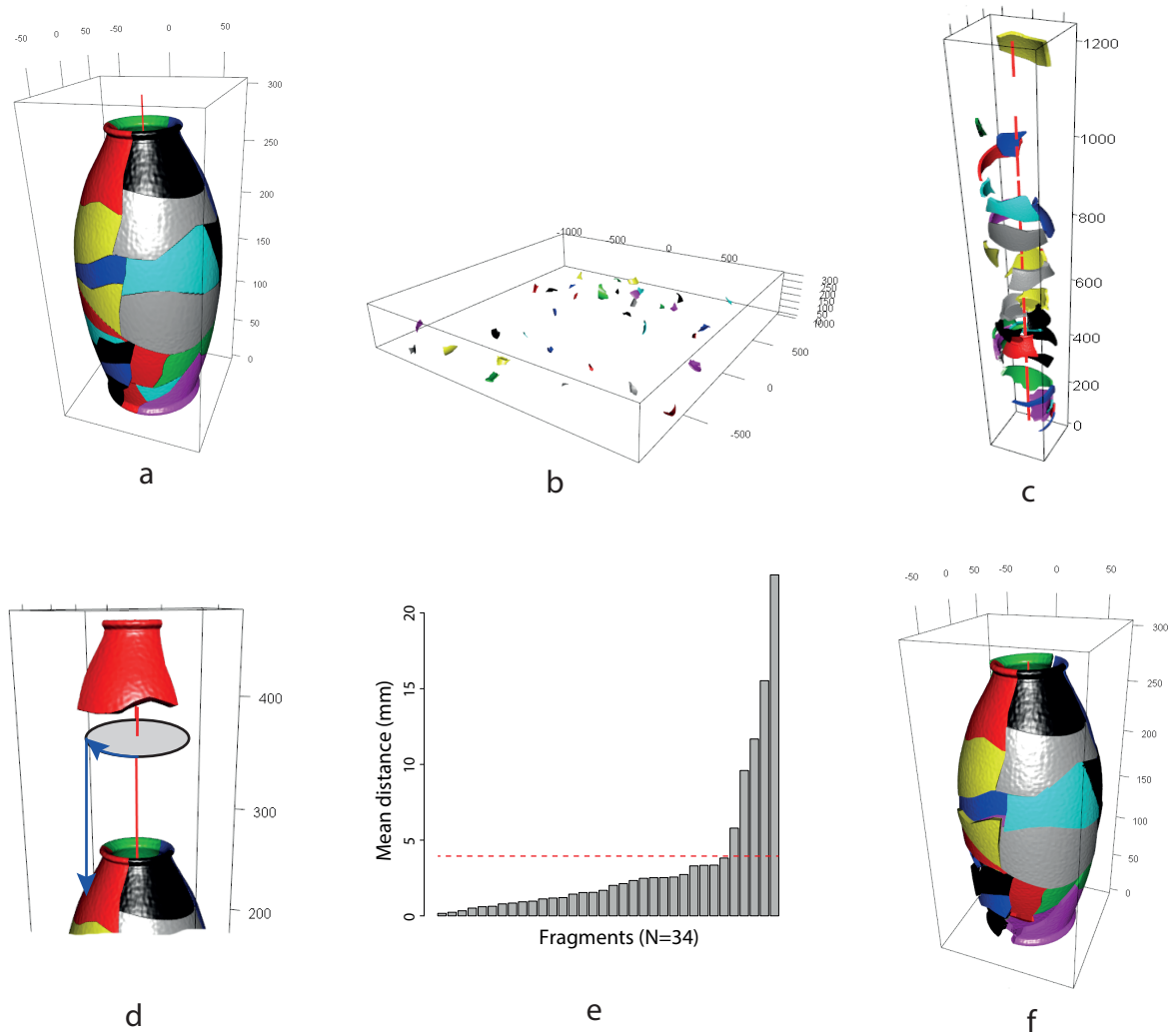


Fig. 8. Evaluation of the automatic workflow on a synthetic vessel. a) Synthetic vessel broken into 34 fragments. b) Randomly translated and rotated fragments. c) Fragments oriented by the system. d) Placing an aligned fragment in its original position on the vessel. e) Mean point-to-point distances between original and aligned fragments. f) Resulting aligned fragments.

minutes, the combination of all these illustration techniques produces results encompassing the most visible aspects of the pottery fragments. The 3D reconstruction of real-world pottery fragments was obtained in a few seconds (see Supplementary materials S5).

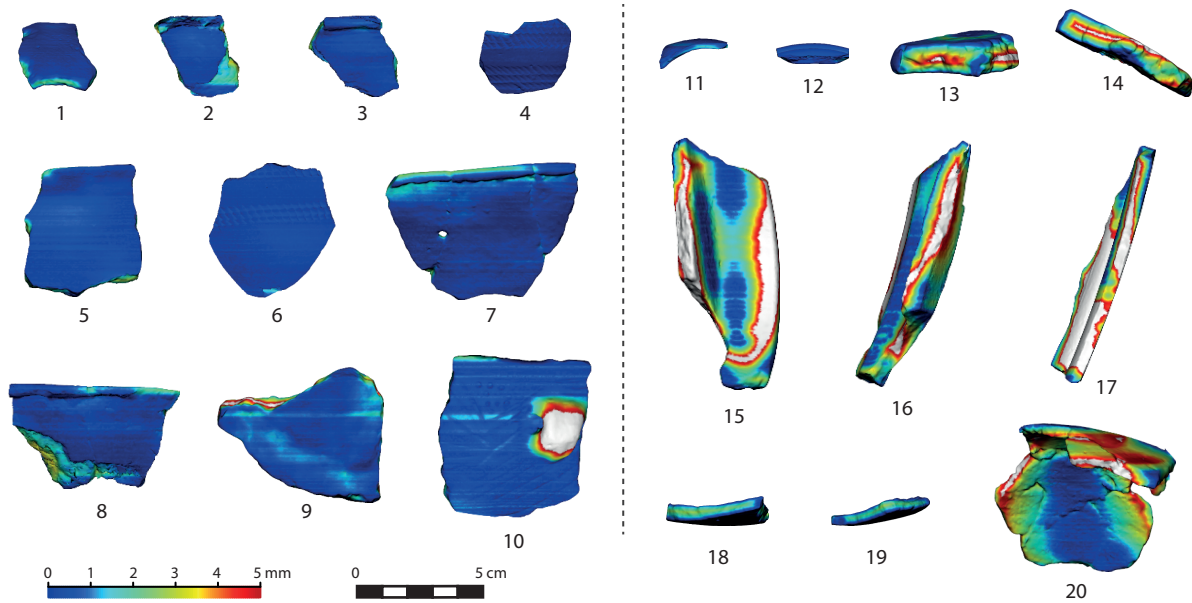


Fig. 9. Evaluation of the automatic workflow on real-world pottery fragments. 1-10) Example of well-oriented fragments. 11-20) Example of incorrectly oriented fragments. See Supplementary materials S3 for more examples.

Table 1. Comparison of the radius of six fragments, determined (i) by an expert using traditional tools (radius expert column) and (ii) by four archaeologists using different procedures of pre-orientation and final adjustment (R1-4 columns). The sector angle provides information about fragment preservation. Range and standard deviation (sd) were calculated for the R1-4 values. Corresponding drawings are available in Supplementary materials S4.

Fragment number	Sector angle	Radius expert (cm)	R1 (cm)	R2 (cm)	R3 (cm)	R4 (cm)	Range (cm)	sd (cm)
3077.22	9°	10.00	9.55	9.10	9.30	9.20	0.45	0.19
3077.35	12°	7.25	7.10	7.15	7.00	7.00	0.15	0.08
3077.42	7°	12.25	-	12.50	-	12.70	0.20	0.14
3107.08	13°	9.25	10.20	10.15	9.90	10.05	0.30	0.13
3107.25	10°	11.00	11.45	11.80	-	11.75	0.35	0.19
3107.26	18°	5.00	5.05	4.95	4.35	4.35	0.70	0.38

4 DISCUSSION

4.1 Strengths and Limitations of Orientation Methods

Although the specialist is usually able to sketch the profile of a fragment quickly, without any 3D modelling and subsequent extra time and cost, it is reasonable to expect that this 3D technology will soon be simplified, and available to the general public, as was the case for photography in its time. The low-cost 3D acquisition device used here is quick to operate (ca. 5 min), and sufficiently accurate for ceramic studies. Generally, fragment

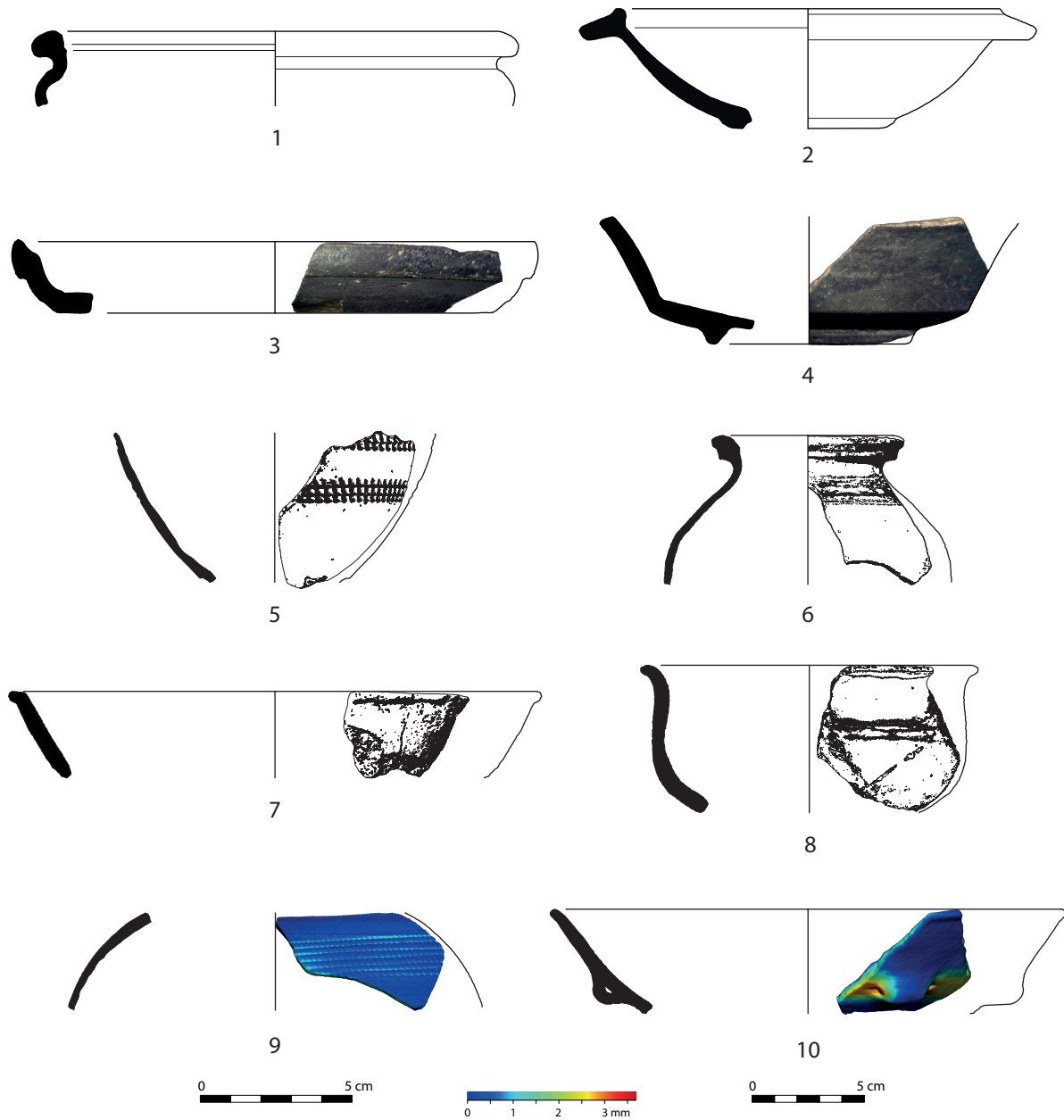


Fig. 10. Examples of archaeological illustrations. 1-2) Linear mode. 2-3) Colour mode. 5-8) Shaded mode. 9-10) Vessel regularity mode.

size does not play a key role in the orientation of fragments, except when they are so small that the number of points is not sufficient for adequate processing. What matters more is the sector angle. Neither flat bases nor spherical fragments can be oriented automatically because all normals are parallel with the rotation axis, or they converge to a single point, producing an infinite number of solutions [34, 65]. Our approach was generally able to find the optimal rotation axis even for irregular hand-turned pottery, and for small fragments that would have been discarded by a skilled archaeologist. This result may considerably increase the quantity of data available for further archaeological interpretation. If enough fragments are well oriented, the possibility of automatic reassembly is greatly increased because their position with respect to the original rotational axis will provide a strong constraint (e.g. [10]). Sometimes the optimal orientation defined by 3D model geometry may differ from that preferred by ceramic specialists (e.g. Fig. 9:1-2, 7-8; Fig. 10:3, 7, 10). This is often true for upper parts, where the rim is traditionally fitted to a horizontal plane, perpendicular to the rotation axis (e.g. [2, 12, 26]). This strategy supposes that the potter created vessels with perfectly regular rims. Nevertheless, even wheel-turned pottery rarely satisfies this requirement, because centrifugal force considerably handicaps the potter's effort to keep the upper part of the vessel perfectly regular. Differences with traditional orientation decisions may also be seen when fragments contain decorations that are not perfectly horizontal, but that would naturally have been used to orient the vessel (e.g. Fig. 9:6; Fig. 10:5). Considering the variability of ancient production and post-depositional processes, every pottery fragment should be considered unique, and thus deserving of special attention. Should symmetry and overall 3D geometry, decoration, or rims be preferred for pottery orientation? The suite of functions proposed here allows all existing orientation solutions to be explored, but the final choice remains that of the archaeologist.

4.2 Quality of Archaeological Documentation

The pottery illustrations conform to current drawing norms. The shading tools can produce a detailed, very realistic, visual representation of the fragment surface, including most of its visible features. The vessel regularity representation provides a reliable estimate of orientation quality, and can also be used to study pottery-making tradition and specialisation. If required, illustrations can be modified in any graphical software. Profiles extracted can also be used for automatic classification of fragments based on their profile geometry (e.g. [17, 24, 29, 31, 35, 55, 59]). Profiles, illustrations and models can also be integrated into existing databases and ongoing projects (e.g. ArchAIDE [19]; GRAVITATE [49]). Adding a third dimension to profiles augments the sense of depth, enhancing visual perception of their original shape, volume and function. The greatest advantages of the suite of functions presented here are the gains in terms of time and amount of information obtained. The aim is for archaeologists to be less hampered by routine tasks. Pottery drawing based on 3D models may advantageously replace traditional tools, such as the stylus, profile gauge, and radius chart. The documentation produced is optimised for sharing (electronically or in paper format), presentation, and archiving. It can be further consulted for morphological, stylistic, or technological classifications. Besides these possibilities, the 3D models can be included in virtual reconstructions, developed in museography, education, or in the entertainment industry. These reconstructions can also be reproduced by 3D printers in a variety of materials (resin, plastic, ceramics, metal, etc.), and serve for exhibition, in place of plaster, as a support medium for the original fragments.

5 CONCLUSION

The suite of functions proposed here allows the expert to choose between several modern computational methods to determine the position of the rotation axis of pottery fragments, and to draw them. It presents two main advantages: (i) significant gains in time, and thus in money invested, and (ii) a fine combination of reproducibility and accuracy, accessible at different skill levels. The entire process, including 3D model acquisition, fragment orientation, and graphical illustration, should not exceed 10 minutes, no matter how complex the fragment may be.

It is not limited to wheel-turned pottery, and can process a vast range of pottery fragments (rims, body parts, and bases), increasing both the quantity and quality of information derivable from the available corpus. The success rate of the fully automatic workflow is 85% on synthetic data, and 77% on real-world archaeological fragments, but reaches almost 100% when problematic fragments are manually pre-oriented. The graphical outputs (in linear, photorealistic, or shaded modes, including scale, preservation indicator, and basic measurements) are in conformity with current standards of archaeological illustration. The graphical features, such as pottery symmetry and irregularities, which cannot be obtained by conventional methods, may provide deeper insight into many aspects of pottery production. The final output can be exported, archived, and shared, allowing further study. The approach is graphical, user-friendly, intuitive, and does not require knowledge about coding. The suite of functions integrated in the application DACORD is available in Supplementary material S1. This tool can thus be freely improved and optimised by the archaeological community. Although the tool was developed in R, it can be adapted to a more rapid programming language and/or implemented in many systems focused on the documentation, classification and reconstruction of archaeological pottery, to improve speed, accessibility, and operability.

A SUPPLEMENTARY MATERIALS

See the supplementary materials in the online version.

ACKNOWLEDGMENTS

We are very grateful to the CNRS (BQR-PRES and PEPs HUMAIN grants) and SATT Grand Est for funding and material support. We also wish to thank the museum of Lodève for allowing us access to their collections, and the master students who helped in scanning. The French Minister of Foreign Affairs is thanked for JW's fellowship. We are also very grateful to Remi Laffont, Gilles Hamm, and the Bibracte Centre Archéologique Européen, and more particularly to its director, Vincent Guichard, for constant support, interest and feedback about the tool. We are grateful to the five anonymous reviewers whose judicious comments have greatly improved the manuscript.

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Received October 2017; revised February 2018; accepted April 2018